Numbers seem to occupy a paradoxical place within our ontology. On the one hand, they are the stuff of everyday life, what we appeal to when we count the days until our birthday, when we add the calories in our diet, when we calculate our taxes, etc.; on the other, they are the stuff of sophisticated studies in technical journals where people try to crack mysteries like the construction of quasirandom points for efficient multidimensional numerical integration. They seem to be an integral part of our concrete and natural world, the sort of thing we cannot do without if we want a fully integrated scientific picture of the world – they are heavily involved in many of our best theories of natural and social science; but they also seem to belong to a sui-generis realm of abstract objects with no spatial location or causal powers. They are both the most familiar and the most esoteric of objects. They are so simple that even non-human animals have been shown to have at least the most basic of numerical abilities, and some have even been shown to be able to perform simple arithmetic operations; but they are so complex that to state even some of the most simple questions has proved to be extremely difficult to answer. They are so common that it seems that every culture in every historical period has been able to discover at least some basic mathematical truths and being able to develop some mathematical skills; yet most of what we know about numbers has been discovered in the last couple of centuries as mathematics became an academic scientific endeavour.

It is not surprising, therefore, that numbers have awoken the curiosity of philosophers throughout history, trying to make sense of these strange, yet familiar, entities. The task, however, has proved to be elusive. It has been extremely difficult to generate an account that integrates all these dimensions of the phenomenon and that is able to harmonize all these tensions: Philosophical accounts of numbers that link them closely to our everyday practices of counting and calculating have had a very hard time explaining their being infinite in number and complex in their structure. In contrast, if we pay too close attention to the many historical contingencies that have shaped their development, we might be tempted to conceive them as artificial constructions of our culture – that "the mathematician is an inventor, not a discoverer" as Wittgenstein famously argued (1978: 99) –; but this would make it very hard to explain why they seem to pop up independently in every human culture or why they are so useful in the formulation of successful theories in the natural sciences. In general, the more we try to ground numbers in one of their many fundamental aspects, the more puzzling others become.

Numbers themselves are a heterogenous bunch. They come in all sizes and shapes. Some are simple and small like two and three, while others are weird like the octanions of string theory, or huge like inaccessible cardinals. Integers like 35 or 2,345,000 are complete in a way that rational numbers like 0.35 or 27/34 are not. Some are finite, others are infinite. Some are real, others are imaginary. Some are fascinating and serve as touchstones of amazing theories, like Pi or *e*, while others are run of the mill and dull, like 1,269,787, while even number 30 can be deemed interesting in so far as it happens to be the largest integer such that all smaller integers with which it has no common divisor are prime numbers. Yet, most mathematicians and philosophers have focused their attention on the so-called *natural* numbers, i.e., 0, 1, 2, 3, 4, etc. It is commonly assumed, but hardly ever fully defended, that if we can account for the nature of these simple numbers, we would have cracked a sufficient entrance into the mathematical real from which we can reach the rest of the numerical world.[1]

Natural numbers are the simplest cardinal numbers, i.e., the simplest (and until recently, the only) numbers we use for counting (finite sets or pluralities). As a matter of fact, it is commonly assumed that natural numbers and counting are so essentially linked that one cannot have one without the other (Comrie 2006). Real numbers, like three and a half, 12.735 or *pi*, in comparison, are not as good for counting – after all, it seems impossible to count up to 3.45 – and work better for measuring magnitudes – hence, I do not say that I am 178 cms tall, but instead say that I am 1.78 m tall.

In close kinship with the issue of how essential is counting to the very nature of cardinal numbers is the question of how arithmetical truths and theorems of mathematical number theory like the fundamental theorem that "every integer greater than 1 either is a prime number itself or can be represented as the product of prime numbers" are related to everyday statements like "Kris Bryant hit a homerun for the sixth time in twelve games last monday". There is a common tendency among contemporary philosophers of mathematics, at least since Gottlöb Frege, to argue that numbers are essentially just what we use to count and that arithmetic and number theory are nothing but the abstract mathematical theories we use to study them. Gómez-Torrente, for example, has argued that

[1]. For a dissenting view, consider Brandom's claim that "Semanticists, metaphysicians, and ontologists interested in mathematics cannot safely confine themselves, as so many have done, to looking only at the natural numbers." (Brandom 1996, §6) apud. Shapiro (2008).

> "… a number *n* has essentially the property of being had by any plurality of *n* things… For example, 17 essentially has the general property of being had by any plurality of seventeen things. And in general, a number *n* essentially has… the purely general property that any plurality of *n* things will have it." (Gómez-Torrente 2015, 318)

This means that the essence of cardinal numbers lies in their relation to countable pluralities. From this essential nature stem the rest of their necessary properties, the sort of properties that we study and systematize in arithmetic and number theory. Thus, from this perspective, as Dedekind wrote in 1872, we can "…regard the whole of arithmetic as a necessary, or at least natural, consequence of the simplest arithmetic act, that of counting…" (Dedekind 1872, §1)[2]

In contrast, more formalist philosophers and mathematicians have argued the contrary, i.e., that the essence of cardinal numbers is to be found not in their relation to pluralities, but in their relation to each other (Shapiro 1997); more in mathematics than in our everyday judgements of cardinality. For Shapiro, for instance, structural properties like, for "…example, the property of being a prime number [tell us more about what a number is, than, say, the] property of being the number of my daughters, or of being one of James Ladyman's favorite numbers…" (Shapiro 2008: 286) For this tradition, as David Fair (1988) has defended, mathematics adequately characterizes natural numbers insofar as they have no "additional properties about their essential nature which mathematicians had never noticed before."[3] (Fair 1988,

[2]. But see Shapiro (1997: 175) for a different interpretation of this passage.

[3]. Except, perhaps, for exactly this, i.e., that numbers have no essential non-mathematical properties (Shapiro 2008: 307-8).

368) Numbers, in other words, are nothing but what number theorists know them to be.

Both traditions face the same sort of challenge, to explain how the same sort of objects, i.e., cardinal numbers, can be both involved in our everyday practice of counting objects, both concrete and abstract, and in the rarified formal practice of mathematics. For the formalist, there is an important ontological separation between the mathematical system and its application to some subject-matter outside pure mathematics (Stenlund 2015, 46) and thus the challenge is to explain how this application is even possible. The Fregean tradition, in turn, faces the inverse challenge: to explain how we get from our everyday practice of counting things to the complexities of arithmetic and number theory. Just as the problem for formalist is to account for the **application** of abstract numbers to concrete instances, the challenge for the Fregean is to account for the inverse process of **abstraction**, i.e., how we get from concrete instances of cardinality to abstract numbers. The challenge is especially difficult in the case of more complex mathematical numbers (Wigner 1960), but it is still there in the case of finite cardinal numbers.

This distinction makes an important epistemological difference. If cardinal numbers are essentially those we use to count, then counting might also have an epistemological preeminence as means to accessing truths about and involving numbers. Hence the importance paid to cognitive studies of counting in developing a naturalistic epistemology of numbers in recent decades (Giaquinto 2014). On the other hand, if numbers are first and foremost what mathematicians study, then the knowledge of numbers is nothing but the knowledge of mathematics.

In the end, no matter what position we assume regarding the relative fundamentality of counting in arithmetic, we need to develop a good account of what happens in cardinal statements. This is obviously true if we think of of cardinality as the essential feature of cardinal numbers, but even if we think that arithmetic is an autonomous abstract science, we still need a good account of its application, and this is going to require a full account of what our cardinal statements like “I left three apples on the table” or “Before I left the party, they had already played *La Tusa* three times” are about. This means that the contents of this book would be of enormous importance for any philosopher of mathematics interested in the nature of arithmetics and its entities.

1. Language, Linguistics and Mathematics

There is a broad area of intersection between contemporary linguistic analysis and mathematics. Not only are mathematical tools used in different areas of linguistics, there is also a considerable strand of linguistic research in which mathematics does not (only) play the role of tool, but also of topic and subject of analysis. Traditionally, the linguistic study of mathematics has come in one of two main flavours depending on whether mathematics is seen as subject or tool for linguistic analysis.

On the one hand, it is not uncommon to consider mathematical formalisms as a sort of technical language, substantially different from the natural languages for which linguistic analysis was designed, but with enough *language-like attributes* to justify a linguistic approach. Just like sentences of natural languages, mathematical formulas seem to be syntactically constructed out of simpler symbols. Just like sentences of natural languages, some of these formulas are said to be true or false. “It is clear that the expression

$$x(^2 \frac{+}{y8-}$$

is syntactically incorrect, while the equation

$$\frac{7^2 - 9}{5} = (3 - 2)^3$$

is syntactically well-formed but not true” (Jansen, Marriott and Yelland, 1999) These are the two most common features supporting the talk of mathematical languages and the use of linguistic tools to analyze mathematical formalisms. Under this perspective, these formalisms are seen as artificial languages, different and even independent of any natural language.

This does not mean that mathematicians do not deploy resources from every-day natural language to explicate the mathematical content of such formulas, in what Wittgenstein called “mathematical prose”. Sentences like “seven is prime” or “the semigroup ideals in a ring are identical with the ring ideals if and only if the semigroup ideals form a linearly ordered set under set inclusion” have a strange status in the twilight zone between mathematical and natural language. They employ what Reuben Hersch (1997) called “mathematical lingo”, i.e., words and syntactic structures from natural language that are borrowed with new, *sui-generis* senses and functions. Like any other technical language is an open debate whether a linguistic theory of natural language ought to account for these uses. This issue will come later, when we compare how numerals are used as part of the mathematical lingo and in ordinary uses. As we will see, the way mathematicians use numerals differs with the way non-mathematicians use numerals both in grammar and semantics.

As another point of contact between mathematics and linguistics, and like many other disciplines, several linguistic fields – most notably, syntax and semantics – have adopted mathematical tools for the analysis and representations of their theories. The history of how linguistics adopted these mathematical tools is convoluted but illuminating. It starts deep within the foundational debates at the turn of the XXth Century. From different perspectives, philosophers and mathematicians were worried about the strange nature of their subject of study and of numbers in particular. As is usual in debates like these, empiricist like Alfred Tarski née Tajtelbaum, following on the steps of Kantian formalists like David Hilbert, tried to develop different accounts of nominalist arithmetics where there were no numbers, only numerals. Coming up with a nominalist foundation for arithmetics required the development of a rigorous formal theory of language. Most of the basic formal tools still used today in the formal study of semantics were thus developed by Polish logicians like Stanisław Leśniewski, Jan Łukasiewicz and the aformentioned Alfred Tarski for the study of formal languages. Yet, the tools proved so powerful that they were soon adapted for the study of natural languages too, starting with the seminal work of Richard Montague, who famously stated that he found no "important theoretical difference ... between formal and natural languages." (Montague 1970) This bold project was further developed by a long generation of logicians and linguists like Jon Barwise and Barbara Partee who, in 1978, argued that, given that at least within the Chomskyan paradigm, the goal of linguistic semantics (and syntax) is to describe "the structure of a certain mental faculty, and mathematics is the best available tool for describing structure" (Partee 1978, 2) we could well see semantics as both a psychological and a mathematical discipline.[4]

2. Numbers and Numerals

Common sense makes an important distinction between world and language. This is not to say that world and language are not intimately linked. After all, much of language and what we do with it seems to be clearly **about** the world.[5] When we say that the sun is shining, for example, we are saying something about the sun and the weather. When we ask someone how they are feeling, we are asking about the actual state of their health. These things – the sun, the weather, our health, etc. – are part of the world, and we use words to talk about them. Yet, this does not automatically mean that every word corresponds in some way to some element or aspect of the world. Some do, some don't. Perhaps the least controversial examples of words that are about something are names. Names essentially name – or, at least, aim at naming – something. On the other hand, there does not seem to be something in the world that words like "neither" or "if" are about. At least since the Thirteenth Century

[4]. This is not to say that the project has not had its many antagonists. In (2009), for example, Pieter A. M. Seuren complained that "Despite the ... claims made by some schools of formal semanticists, their work has, in actual fact, very little to do with the mind and everything to do with the development of new, sophisticated subtheories within the overall structure of standard logic and mathematics. The mind is, in other words, merely a playground providing an excuse for mathematical and logical diversions." (Seuren 2009, 4)

[5]. We need to understand 'the world" here widely enough to cover not just the world as it actually is, but also as it might be, as we would want it to be, etc. (Barceló forthcoming a).

philosophers have considered these so-called *syncategorematic* words as having a different sort of function in language that being about something in the world. It is an interesting open question whether number words are closer to names or to syncategorematic words in this regard, i.e., are they about something in the world? – and if so, what? – or do they have an altogether different function? – and if so, which one?

Nevertheless, if a number word like "seven" corresponds to anything in the world, it must surely correspond to number seven. But exactly what does this relation of 'correspondence' ammount to is a very difficult question we will tackle in depth in the third chapter. Yet, before tackling this issue, it is important to sharpen our intuitive distinction between numbers and number words. Independently of whether there really are numbers or only number words and other number representations), the simplest way to make the distinction is by appealing to the method of category mistakes.

Category mistakes are a common linguistic technique used in ontology to determine whether terms refer to entities in different categories or not. The basic intuition behind this technique is that, just as entities can be classified in different ontological categories – concrete or abstract, animated or inanimated, universal or particular, etc. – properties can also be classified depending on what sort of entities they apply to. Thus, we can classify entities in categories by identifying which properties are properly predicated of them and which are not. For example, sentences (1) and (3) make perfect sense, while sentences (2) and (4) not – they express what is not known as "category errors" – because, presumably, while "John's arrival" refers to an event, "the building" refers to an object. Objects and events are different ontological categories, objects are the kind of thing that can *be there* at a certain time, while events are the kind of thing that can *take place* at a certain time. (Moltmann 2017, Thomasson 2019, Barceló forthcoming b)

(1) John's arrival took place last week.

(2) The building took place last year.

(3) The building was already there last year.

(4) John's arrival was already there last week.

The same method can be applied to tell numbers and numerals apart. Consider the following examples:

(5) Seventy is even.

(6) "Seventy" rhymes with "enemy".

Sentence (5) is about a number – number seventy, while sentence (6) is about a number word – "seventy". Seventy, as a number, can be odd or even, large or small, or divisible by seven or not, etc. "Seventy", the word, certainly cannot. It just does not make sense to say of a number word that it is odd or even, that is divisible by seven or not, etc. Words like "seventy", on the other hand, have syntactic and phonological properties, they can certainly be short or long, start with a vowel or with a consonant, be in English or in Ainu, rhyme with "foe" or with "enemy", etc. Numbers, however, are not the sort of things that can have any of these later properties: they do not contain letters of phonemes, they do not belong to languages and therefore, they cannot rhyme or not rhyme with a word or other.

This distinction is usually cashed out also in the distinction between **using** and **mentioning** a word. In the first sentence, the sentence about the number seventy, the word "seventy" is used to talk about that number, seventy; in the second, it is just mentioned. Sometimes it is said that the word “seventy” is used to talk about itself in sentences like (6). We also often use quotation marks around the word being mentioned to mark this distinction.

As intuitive as this distinction goes, however, things are not that simple. Consider sentences (7) and (8):

(7) Steffie has just learned to count up to seventy!

(8) Seventy is my favourite number.

It is not starightforward to determine whether, in these sentences, “seventy” is being used to refer to a number or instead the sentence is talking about the number word “seventy”. When we learn to recite the numerical sequence as kids, are we learning something about numbers or just about the words that we use to refer to them in our language? As we have mentioned, it is usually defended that cardinal numbers are essentially linked to cardinality and the process of counting, however it is not clear whether and how does learning the numerical sequence is related to the process of counting items in a group. Similarly, it is also not obvious whether and when what we like – or think about, or imagine, etc. – is a number or a numeral. It is not the same to like seventy because it contains a seven, than to like it because it is divisible by seven, for example.

Consider now sentence (9):

(9) Seventy people showed up to the party.

Is (9) about number seventy, just as sentence (5) or not? It certainly does not seem to be about the number word “seventy”, but it is also not obvious that it is about number seventy either. What is happening here, then?

For centuries, philosophers have tried to answer this very complex question and their proposed answers have usually involved some form of linguistic analysis. It is usually said that the main difference between (1) and (3) is that "seventy" functions as a nominal in (1) but not in (3). However, as well will see in due course later in the book, even if it is not a nominal, it is hard to tell exactly to what grammatical category "seventy" belongs in (3): is it an adjective, or is it a determiner similar to a quantifier? And if so, what does this tell us about numbers themselves?, does this show that numbers are not objects, but properties, or processes?

Furthermore, even if we accept that numbers and numerals are entities of different sorts, this still does not tell us what sort of entities numbers are. In other words, even if it is fairly straightforward to say that numbers and numerals belong to different ontological categories, this still does not tell us to what ontological category numbers belong. As we have mentioned at the beginning of this chapter, numbers are the kind of entities that have arithmetical properties like being divisible by some numbers, but not others, being larger than some numbers, but smaller than others, etc. However, besides these basic and necessary properties, it is difficult to say much more about them. Indeed, if we apply the aforementioned technique of finding category errors in how we talk about numbers, it is very difficult to say much about what numbers are, yet we might find much about what numbers are **not** (Lewis 1986: 82-84)! Consider the following sentences (10) - (17), they all seem to be category errors:

(10) Seventeen took place last year.

(11) Seventeen was already there yesterday.

(12) Seventeen is here.

(13) I had not noticed seventeen.

(14) Seventeen was not as heavy as I thought.

(15) Seventeen did it!

(16) Seventy was divisible by seven.

(17) Seventy will be divisible by seven.

(18) The rain did not cause seventeen.

(19) Seventeen did not cause the rain.

etc.

Presumably, this means that numbers are neither events nor causally linked to events, they are not spatio-temporal or material, they are not agents, etc. But again, this tells us very little about what sort of entities numbers are! Metaphysicans of all times have battled with this question for ages. For some, from Plato to James Robert Brown, numbers are the inhabitants of an abstract realm. For others, like Stuart Mill, they are universals, corresponding to properties of groups (Mill *System*, VII: 254). Others, like Hartry Field, take them to be fictions ,while others, like Gottlöb Frege, Bertrand Russell, Crispin Wright or Robert Hale take them to be logical entities.

Furthermore, as we have seen at the beginning of this chapter, besides the aformentioned arithmetical properties, numbers are also presumed to have three other problematic properties: first, they are easily named by number words and other representational devices, like numerals; second we know things about them, like that they are infinite or that some of them are bigger than others and, finally, they are usually fruitfully applied to all matters of human interest, from the explanation of natural phenomena to the distribution and exchange of economic resources. How is that possible, given their sui-generis ontological status? In other words, how is it possible for us, mortal, concrete, material beings, to know, name and apply numbers so well and to such advantage to us?

In the previous section, we drew a distinction between two ontological traditions in the philosophy of numbers, based on whether the essence of numbers is found in mathematics our in our everyday judgements of cardinality. This distinction also makes an important epistemological difference. If cardinal numbers are essentially those we use to count, then counting might also have an epistemological preeminence as means to accessing truths about numbers. Hence the importance paid to cognitive studies of counting in developing a naturalistic epistemology of numbers in recent decades (Giaquinto 2014). On the other hand, if numbers are first and foremost what mathematicians study, then our epistemology must make use of other tools, from formal logic to ethnology, better suited for modelling and analyzing our scientific mathematical practices.

Each tradition offers a different account of how we access numbers. For the first tradition, gaining cognitive access to numbers is nothing but knowing how to use numbers to make judgments of cardinality. As aforementioned, counting is presumed to place a central role in this process.

> “According to the second account, natural number concepts depend on a specific product of culture: a counting procedure. These concepts are recent and unique to humans, because the first counting procedure appears to have been invented relatively late in human prehistory. They are learned: indeed, contemporary children master counting procedures slowly and with difficulty. And they are culturally variable: different human groups count in different

> ways and to different extents, and some groups do not count at all." (Spelke 2017, 148)

This means that different counting practices will give rise, and epistemic access, to different numerical systems. As Bernard Comrie has argued,

> "Speakers of languages with restricted [numeral] systems, such as Australian languages [Mangarayi and Yidiny], typically did not engage traditionally in counting. The number of entities was arrived at by "subitizing", i.e. immediately recognizing the number, as is possible up to around 5." (Comrie 2006)

This means that peoples who do not count do not develop full fledged numeral systems. We have the capacity to directly detect the cardinality of very small groups without counting them. This phenomenon is commonly known as perceptual subitizing, and most likely grounds our other arithmetic capacities. Still, by itself, subitizing is not enough to deliver cognitive access to cardinal numbers larger than four. In consequence, it can be only a partial ground of our capacity to think about numbers at most.

Thus, for this tradition, counting is a necessary step in developing any epistemic access to numbers, but it is not sufficient. Something else seems to be needed and natural language competence is an attractive hypothesis. According to Spelke, "... the development of natural number concepts depends on the acquisition and use of a natural language." (2017)

This approach to numerical knowledge contrasts to more traditional epistemologies based on formal proof and calculation, from Frege and Hilbert's seminal work at the turn of last Century to the more recent proposals from philosophers like James Robert Brown, Stewart Shapiro and Penelope Maddy. The main challenge faced by these more formalist epistemologies is trying to explain how it is possible for someone to accede to abstract, universal and eternal mathematical truths by scribbling on a notebook or blackboard. It seems natural to expect objects to play some role in how we gain knowledge about them, yet numbers by their very abstract nature do not seem to be the sort of entity that can interact with humans like us. The explanation of human mathematical knowledge has thus proved to be an enormous philosophical challenge.

Given the many difficulties in trying to determine what sort of entities numbers are, and what place they occupy in our overall picture of the world, many philosophers of the last centuries have opted to exclude them from their ontologies. In other words, they have defended the view that numbers *just don't exist*.

Arguments in favour of the existence of mathematical entities like numbers can be classified into two broad kinds: on the one hand, we have those that start from mathematical practice as a given and postulate the existence of mathematical objects as part of the best explanation for such practice being as its. Usually, this is either because of its success as a scientific enterprise or because of its importance for other successful scientific and technological practices. On the other hand, there are those that, instead, take everyday linguistic practice as starting point and postulate the existence of mathematical objects as part of the best explanation of our using natural languages as we do. The overall general strategy in both cases is to argue that without numbers, it would be very hard to explain why things that we accept to be true (or, at least, to be successful as claims about the world), i.e., simple arithmetical truths like seventeen being prime, complex physical laws like the superposition principle or just everyday assertions like there being twelve judges in the Supreme Court – are actually true. The basic idea is that something cannot be true unless the things it is *about* actually exist, and that these truths are, at least in part, about numbers. Thus, given that if we know what something is true and that it is about some category of things, then we have good reasons to conclude that such things do exist, we must hence conclude that numbers exist. Bob Hale, for example, has argued:

> If entities belonging to a certain ontological category just are what expressions of a certain logical category stand for, then we can argue for the existence of entities of that kind by arguing that there are true statements involving expressions of the relevant kind. If, for example, there are true statements incorporating expressions functioning as singular terms, then there are objects of some corresponding kind. If the singular terms are such that, if they have reference at all, they refer to numbers, there are numbers." (Hale 2010, 406; quoted by Thomasson 2014, 133)

This line of reasoning brings forth the importance of linguistic analysis for the ontological enterprise of determining what sort of things conform our reality and, in particular, whether numbers do indeed exist. If Hale's argument carries any force, the question of whether there are (literally) true sentences where numerals function as singular terms would have enormous ontological importance![6]

[6]. Thanks to the influential work of Dummett (1991), the hypothesis that if numerals in natural language are singular terms, then numbers exist is commonly associated with Frege. Nevertheless, closer reading of Frege's texts shows that the German philosopher was not interested in natural language, but the many representational means mathematicians employ in their work. The sense-reference distinction, for example, is usually seen as a semantic distinction that applies to expressions in natural language; however, when Frege introduces it in his *Conceptual Notation* (1879) – before he used the terms "sense" and "reference" – he uses as illustration an example of diagrammatic Geometry.

References

Barceló, Axel (forthcoming a) "Condiciones de Verdad".

Barceló, Axel (forthcoming b) *Introducción a la Ontología*.

Bernard Comrie (2006) 'Typology of Numeral Systems" in *Numeral Systems of the World's Language,* URL = https://mpi-lingweb.shh.mpg.de/numeral/TypNumCuhk_11ho.pdf

Dedekind, R. (1872) "Stetigkeit und irrationale Zahlen", Brunswick, Vieweg; translated as "Continuity and irrational numbers", in *Essays on the theory of numbers*, edited by W. W. Beman, New York, Dover, 1963, 1–27.

Brandom, Robert (1996) 'The significance of complex numbers for Frege's philosophy of mathematics", *Proceedings of the Aristotelian Society* 96, 293–315.

Dummett, Michael (1991) *Frege: Philosophy of Mathematics,* Harvard University Press

Fair, David (1984) "Provability and Mathematical Truth", *Synthese* 61: 363.

Frege, Gottlöb (1879) *Begriffsschrift, eine der arithmetischen nachgebildete Formelsprache des reinen Denkens*, Halle a. S.: Louis Nebert; translated as "Concept Script, a formal language of pure thought modelled upon that of arithmetic", by S. Bauer-Mengelberg in J. van Heijenoort (ed.), *From Frege to Gödel: A Source Book in Mathematical Logic, 1879–1931*, Cambridge, MA: Harvard University Press, 1967.

Giaquinto, Marcus (2014) "Philosophy of Number", *The Oxford Handbook of Numerical Cognition*, edited by Roi Cohen Kadosh and Ann Dowker, 17–34.

Gómez-Torrente (2015) “On the Essence and Identity of Numbers”, *Theoria* 30/3: 317-329

Hale, Robert 2010

Hersch, Reuben (1997)

Jansen, Anthony R., Kim Marriott and Greg W. Yelland, (1999) “Perceiving Structure in Mathematical Expressions”, *Proceedings of the Twenty First Annual Conference of the Cognitive Science Society,* 229 - 233.

Lewis, David (1986) *The Plurality of Worlds,* Wiley-Blackwell

Mill, Stuart (1963–9) *The Collected Works of John Stuart Mill*, edited by John M. Robson, Toronto: University of Toronto Press, London: Routledge and Kegan Paul.

Moltmann, Friederike, (2017), “Natural Language Ontology”, in *The Oxford Research Encyclopedia of Linguistics*, New York: Oxford University Press.

Montague 1970

Partee, Barbara (1978), “Semantics: mathematics or psychology”. In R. Bäuerle, U. Egli and A.v. Stechow (eds.) *Semantics from Different Points of View*, Berlin: Springer, 1-14.

Seuren (2009)

Shapiro, Stewart (1997) Philosophy of Mathematics: Structure and Ontology, Oxford University Press

Shapiro, Stewart (2008) Identity, Indiscernibility, and ante rem Structuralism: The Tale of *i* and –*i*, *Philosophia Mathematica* 3(16): 285–309.

Spelke, Elizabeth S. (2017) Core Knowledge, Language, and Number, *Language Learning and Development*, 13:2, 147-170, Stenlund, Sören (2015) "On the Origin of Symbolic Mathematics and Its Significance for Wittgenstein's Thought" N*ordic Wittgenstein Review* 1: 7-92.

Thomasson, Amie, "Categories", *The Stanford Encyclopedia of Philosophy* (Summer 2019 Edition), Edward N. Zalta (ed.), URL = <https://plato.stanford.edu/archives/sum2019/entries/categories/>.

Thomasson 2014

Eugene Wigner (1960) "The Unreasonable Effectiveness of Mathematics in the Natural Sciences", *Communications in Pure and Applied Mathematics*, 13: 1.

Wittgenstein, Ludwig (1978) Remarks on the Philosophy of Mathematics