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Grammatical Necessity in Wittgenstein's Middle Period

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I. Introduction

This article recapitulates several arguments to defend Wittgenstein's view during the middle period of his philosophical career[1] that mathematical statement ultimately belong to grammar. In particular, it aims at debunking those arguments based on the mistaken premise that statements of grammar describe the usage of words and, as such, cannot express necessary propositions. Thus, the overall strategy of this work is to demonstrate, first, that statements of grammar, at least under Wittgenstein's notion, do not *describe* the usage of words, and second, that Wittgenstein's grammatical account of mathematics can accurately account for mathematical necessity.

[1]. Wittgenstein's middle period ranges from his return to Cambridge, early in 1929, to 1933. In this point, I side with Dale Jacquette, who wrote, in the introduction to his *Wittgenstein's Thought in Transition*, "I designate [the middle period] from 1929 to 1933. . . The dates are significant and by no means arbitrary. . . In 1930, Wittgenstein began lecturing at Cambridge University. The end of the transition period can be dated approximately to 1933, because Wittgenstein's lectures from this term recorded in the Blue Book, together with the *Brown Book* of 1934, already contain his new methodology and nearly all of the central ideas of his later philosophy as they were to appear in the *Philosophical Investigations*." (West Lafayette: Purdue University Press 1998), 9.

The first part of this article deals with the Quine/Carnap debate on so-called 'truth by convention' and its relevance to Wittgenstein's grammatical account of mathematics. Since Carnap asserted that he developed his view of mathematics as syntax inspired on Wittgenstein's views on the same subject, taking a stance regarding the Quine/Carnap debate on this issue is crucial for the purposes of this article. It is also indispensable to clarify whether or not Wittgenstein held a view like the one Carnap championed, as is defending him against Quine's criticisms.

The second part assembles a defense of Wittgenstein's grammatical account of mathematical necessity out of arguments from Morris Lazerowitz's "Necessity and Language", Zeno Vendler's "Linguistics and the *a-priori*" W. E. Kennick's "Philosophy as Grammar"[2], and J. Michael Young's "Kant on the Construction of Arithmetical Concepts."[3] These arguments show that the objections raised against Wittgenstein equivocate on the meaning of the adjectival phrase 'of grammar'.

II. Wittgenstein's Grammatical Account of Mathematical Necessity and 'Analyticity'

A. Brief Historical Background

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[2]. W. E. Kennick, "Philosophy as Grammar" in G. E. Anscombe & M. Lazerowitz: *Ludwig Wittgenstein, Philosophy and Language* (Bristol: Thoemmes Press, 1996)

[3]. J. Michael Young, "Kant on the Construction of Mathematical Concepts" *Kant-Studien* 73 (1982): 17-46. Michael Young's article differs from those of Vendler, Lazerowitz and Kennick's in that it focuses on questions in the philosophy of mathematics. Michael Young uses an argument similar to the present one to "show that Kant is right in thinking that to ground *a priori* judgements, at least in arithmetic, upon ostensive constructions" is possible [p. 17]. Kant's and Wttgenstein's position regarding the construction of arithmetical concepts differ primarily because the rules of calculation that Kant refers to as 'the universal conditions of construction', are distinct from the concepts whose constructions they govern. *Cf. Ibidem* 28, 29.

According to the middle Wittgenstein, internal descriptions ascribe essential properties to objects, while external descriptions ascribe accidental properties.[4] A description is internal if the concept in the subject includes or implies the concept in the predicate. This characterization of internal descriptions is very close to one of Kant's definitions of analytic judgements. Since Wittgenstein also includes mathematical statements among internal descriptions, this commits him to believe that mathematical statements are somehow analytic.

Before Kant's classical account of analyticity, Locke had already distinguished two kinds of analytic propositions. In *An Essay concerning Human Understanding* [pp. 306, 308], he distinguished between 'trifling' and 'predicative' propositions. Trifling propositions have the form '$a = a$', in which "we affirm the said term of itself." In predicative propositions, "a part of the complex idea is predicated of the name of the whole." For Locke, mathematical propositions are not analytic in either of these senses. After Locke, Kant added a new account of analyticity to Locke's notion of trifling proposition. For Kant, an analytic judgement is (*i)* one whose subject contains its predicate, or (*ii*) one whose negation is a logical contradiction. By offering these two different accounts, Kant laid the foundations for what became the two main doctrines of analyticity in modern western philosophy.[5] For Kant, a judgement is analytic if the subject contains the predicate. However, he allows for two possible interpretations of this 'containment': what Jerrold J. Katz in *The New Intentsionalism* calls 'logical-containment' and 'concept-containment'.[6] Kant's notion of analyticity fused these two notions, and they remained so until the seminal work of Frege. For Frege, Kant's account of

[4]. *Philosophical Remarks* §94

[5]. They have also been called the 'logicist' and the 'idealist' doctrines.

[6]. Jerrold J. Katz, *Mind*, New Series,. CI, 404 (1992) 691.

analyticity in terms of conceptual containment was a psychologistic error. In *The Foundations of Arithmetic* §3, Frege defines analyticity as 'being a consequence of logical laws plus definitions without scientific assumptions.' Wittgenstein's account of logical necessity in the *Tractatus* follows Frege away from the conceptual path and into logicism. This path leads from Wittgenstein directly into the Quine/Carnap controversy.

It is also worth mentioning that at the end of his 1944 article on 'Russell's Mathematical Logic' Kurt Gödel distinguished two senses of 'analyticity'.

> As to this problem [if (and in which sense) mathematical axioms can be considered analytic], it is to be remarked that analyticity may be understood in two senses. First, it may have the purely formal sense that the terms occurring can be defined (wither explicitly or by rules for eliminating them from sentences containing them) in such a way that the axioms and theorems become special cases of the law of identity and disprovable propositions become negations of this law. . .
>
> In a second sense a proposition is called analytic if it holds "owing to the meaning of the concept occurring in it", where this meaning may perhaps be undefinable (i.e., irreducible to anything more fundamental). [Note 47. The two significations of the term 'analytic' might perhaps be distinguished as tautological an analytic.][7]

According to Carnap, Wittgenstein's *Tractatus* endorsed the view of mathematical propositions as analytic in the tautologous sense. However, by the beginning of the thirties, Wittgenstein's view on the analyticity of mathematics had evolved from a purely formal notion into Gödel's second sense.

After the *Tractatus*, considerations about color and the nature of space changed Wittgenstein's mind about the logicist's path. In the *Tractatus*, he had maintained

[7]. Kurt Gödel,. *Works. English & German Collected Works*. Edited by Solomon Feferman et al. (New York: Oxford University Press, 1986) 139

that "there is only *logical* necessity" [6.375]. However, by the late twenties, he could hardly see how the *Tractatus*' logical necessity could account for the necessity of such propositions as 'The blue spot is not red at the same time'. In the early thirties, the notion of grammatical necessity had become a substitute for the *Tractatus*' 'logical necessity'.

B. Carnap

Despite their alleged mutual personal dislike, Carnap always recognized Wittgenstein's influence on this and other philosophical matters. In his "Intellectual Autobiography," Carnap states that "Wittgenstein was perhaps the philosopher who, besides Russell and Frege, had the greatest influence on my thinking."[8] From Carnap's own appraisal, the sources of this influence were triple: (*i*) careful and intense reading of the *Tractatus* by the Vienna Circle, (*ii*) personal contact between Carnap and Wittgenstein from the Summer of 1927 to the beginning of 1929, and (*iii*) "Waismann's systematic expositions of certain conceptions of Wittgenstein's on basis of his talks with him."[9]

According to Carnap,

> The most important insight I gained from his [Wittgenstein's] work was the conception that the truth of logical statements is based only on their logical structure and the meaning of terms. Logical statements are true under all conceivable circumstances; thus their truth is independent of the contingent facts of the world. On the other hand, it follows that these statements do not say anything about the world and thus have no factual content.[10]

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[8]. Rudolf Carnap "Intellectual Autobiography" in Paul Arthur Schillp ed., *The Philosophy of Rudolf Carnap* (La Salle, Illinois: Open Court Pub. Co., 1963) 46

[9]. *Ibidem* 28

[10]. *Ibidem* 25

From Wittgenstein, Carnap received the idea that logical truths are tautologies. In the *Tractatus*, Wittgenstein unsuccessfully argued for the tautologous nature of logical truth for the first time in the history of logicism.

However, the issue of logical truth is both the source of the main agreement and most important divergence between Carnap and Wittgenstein. Michael Friedman has already pointed this out in his “Carnap and Wittgenstein’s *Tractatus*”, where he writes:

> This conception of the tautologous character of logical and mathematical truth represents Carnap, the most important point of agreement between his philosophy and that of the *Tractatus*. But there is also an equally important point of fundamental disagreement. Whereas the *Tractatus* associates its distinctive conception of logical truth with a radical division between what can be said and what can only be shown but not said a division according to which logic itself is not properly an object of theoretical science at all – Carnap associates his conception of logical truth with the idea that logical analysis, what he calls “logical syntax,” is a theoretical science in the strictest possible sense.[11]

In terms of the middle Wittgenstein, the main point of divergence between Carnap and Wittgenstein was the autonomous character of mathematics and grammar. Under the influence of Frege and Russell, Carnap was convinced of “the philosophical relevance of constructed language systems.”[12] During his years in the Vienna Circle, Otto Neurath nurtured Carnap’s idea that a descriptive science of the structure of language – what would become the “Logical Syntax of Language” – was possible. Finally, Carnap’s study of Hilbert and his continuous talks with Tarski and Gödel convinced him of the philosophical power of meta-mathematics.

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[11]. Michael Friedman, “Carnap and Wittgenstein’s *Tractatus*” in William W. Tait. ed. *Early Analytic Philosophy. Frege, Russell, Wittgenstein. Essays in Honor of Leonard Linsky.* (Chicago: Open Court, 1997) 20

[12]. (Carnap 1963, 28)

By the time he had developed his theory of logical syntax, virtually all connection with Wittgenstein's notion of tautology and analyticity seemed lost.[13] Most strikingly, Carnap's logical syntax of language, unlike Wittgenstein's grammar, had lost its autonomy.

Carnap conceived of philosophy as a descriptive, scientific enterprise geared towards formulating the logic of science in a precise meta-language.[14] Instead of an indescribable, but displayable grammar, Carnap expresses his logical syntax in its own object language. Carnap uses Gödel's arithmetization method to embed the syntactic meta-language in the object language (provided that the object language includes elementary arithmetic), allowing it to express its own syntax. However, it immediately follows from Gödel's work that, for a language containing classical arithmetic, 'truth' is a non-arithmetical predicate and thus, not definable in the language itself. Carnap understood this and, consequently, qualified his remarks on this method in *Logical Syntax*. Commenting on the Wittgenstein-Carnap connection, Michael Friedman interprets this as a point in favor of Wittgenstein's autonomous grammar over Carnap's logical syntax.[15]

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[13]. (Friedman 1997, 23)

[14]. Rudolf Carnap, *The Logical Syntax of Language* (New York: Harcourt, 1937) §73.

[15]. "Carnap, characteristically, has transformed an originally philosophical point into a purely technical question – in this case, the technical question of what formal theories can or cannot be embedded in a given object language. Considered purely as a technical question, however, the situation turns out to be far more complicated than it initially appears. . . For it turns out, again as a consequence of Gödel's researches, that it is as a matter of fact not possible in most cases of interest to express the logical syntax of a language in Carnap's sense in the language itself. . . Thus, the logical syntax in Carnap's sense for a language for classical mathematics can only be expressed in a distinct and essentially richer metalanguage; the logical syntax for this metalanguage can itself only be expressed in a distinct and essentially richer meta-metalanguage; and so on. . . Does this same situation does not represent the kernel of truth – from Carnap's point of view, of course – in Wittgenstein's doctrine of the inexpressibility of logical syntax?"

The failure of Carnap's attempt to syntactically define analyticity is a point in favor of the autonomy of mathematics. Carnap followed Wittgenstein in his search for mathematics among the grammatical rules of language. However, while natural language grammar contains embedded mathematical calculi, Carnap was wrong in believing that mathematics' job is to describe this application in a formal meta-language. For Wittgenstein, in contrast, mathematics is autonomous. Every calculus is its own internal application. As such, it does not require an external description or a meta-mathematical formulation.

It is possible to describe a calculus' external application in a meta-language. However, this description is *not* the calculus itself. Describing a syntax is substantially different from calculating. Unlike calculation, description is not autonomous. The truth of a descriptive proposition point outside the description itself. Calculation determines the correctness of its own propositions. The mere description of a calculus' external application cannot fully determine the correctness or incorrectness of its propositions. Gödel showed that Carnap's attempt failed technically, while Wittgenstein showed that the project was also philosophically inadequate.

C. Quine

1. Two Dogmas and the Analytic Nature of Grammar

> The linguistic doctrine of logical truth is sometimes expressed by saying that logical truths are true by linguistic convention. (Quine 1963, 391)

The analytic/synthetic distinction has a long history in modern philosophy. According to Quine's "Two Dogmas of Empiricism", the writings of Leibniz,

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(Friedman 1997, 35-36)

Hume and Kant foreshadow the contemporary distinction. However, both Hume’s “relations of ideas” and Leibniz’s “truths of reason” are quasi-psychological notions. It was Kant who first inserted language at the core of the philosophical characterization of analyticity. The idea of ‘truths independent of fact’ precedes Kant. Nonetheless, starting with him, these truths became also ‘true by virtue of meaning’. Thus, it could be said that the current notion of ‘analyticity’ originates in Kant. After the seminal work of Frege, analyticity secured a central place in contemporary philosophy of logic and mathematics. The discussion of analyticity in this century has grown largely from his conception. Nevertheless, Quine offered the principal arguments against the analytic/synthetic distinction, not in response to Frege, but in response to Carnap’s *The Logical Syntax of Language*.” Those arguments are so convincing that even today a large number of philosophers and mathematicians consider some of the points made in these seminal writings settled matters. For example, Paul A. Boghossian, starts his 1996 article ‘Analyticity Re-considered’ with the following remarks:

> This is what many philosophers believe today about the analytic/synthetic distinction: In his classic early writings on analyticity — in particular, in “Truth by Convention,” “Two Dogmas of Empiricism,” and “Carnap and Logical Truth” — Quine showed that there can be no distinction between sentences that are true purely by virtue of their meaning and those that are not. In so doing, Quine devastated the philosophical programs that depend on he notion of analyticity — specifically, the linguistic theory of necessary truth . . . Now, I do not know precisely how many philosophers believe all of the above, but I think it would be fair to say that it is the prevailing view.[16]

Quine’s strategy against the analytic/synthetic distinction is stunningly novel and elegant. It targets its putative linguistic dimension through the syntax/semantics distinction. For Quine, if some propositions are true in virtue of linguistic conventions, then either their syntax or their semantics determines their truth. In “Two Dogmas,” he distinguishes between ‘*logically true*’ (syntactic) and others

[16]. *Nous* XXX, 3 (August 1996).

(semantic) analytic statements.[17] According to Quine, both the proof theoretical and model theoretical approaches to necessity can only account for analytic statements of the first kind. For the rest of the article, Quine attacks different attempts – mostly Carnap's – at reducing analytic sentences of the second class to those of the first class. According to Quine, Carnap's account of analyticity is unsuitable, because it tries to reduce semantics to syntax. For Quine, 'analytic' is an irreducible semantic notion. He finds no non-circular, suitable, syntactic account of analyticity. Semantics is simply irreducible to syntax.

Wittgenstein's account of analyticity is not semantic, but syntactic. However, it does not correspond fully to Quine's notion of logical truth. Quine's definition of logical truths reformulates Yehoshua Bar-Hillel's reconstruction of Bolzano's definition of analytic proposition.[18]

> First, we suppose indicated, by enumeration if not otherwise, what words are to be called logical words; typical ones are 'or', 'not', 'if', 'then', 'and', 'all', 'every', 'only', 'some'. The logical truths, then, are those true sentences which involve only logical words *essentially*. What this means is that any other words, though they may also occur in a logical truth (as witness 'Brutus', 'kill', and 'Caesar' in 'Brutus killed or did not kill Ceasar'), can be varied at will without engendering falsity.[19]

As a matter of fact, Wittgenstein's grammatical method is indeed very similar to one of the attempts at defining analyticity syntactically discussed in "Two

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[17]. W. V. O. Quine, "Two Dogmas of Empiricism" *The Philosophical Review* LX, 1 (January 1951a) 23.

[18]. "If we suppose a prior invertory of *logical* particles, comrpising 'no', 'un-', 'not', 'if', 'then', 'and', etc., then in general a logical truth is a statement which is true and remains true under all reinterpretations of its components other than the logical particles." *Ibidem* 23

[19]. *W. V. O. Quine, From a logical point of view*, 2nd edition. (New York: Harper books, 1963) 387

Dogmas”. In section III, Quine discusses the account of analyticity, according to which (*i*) “any analytic statement could be turned into a logical truth by putting synonyms for synonyms”[20] and (*ii*), (cognitive) synonymy[21] is “interchangeability *salva veritate* everywhere except within words.”[22] According to him, the main flaw of this latter account is that interchangeability *salva veritate* fails to capture cognitive synonymy. It only captures co-extensionality. In consequence, not only analytic truths, but also synthetic truths may be transformed into logical truths through *salva veritate* substitution. For example, since the current president of Mexico in August 2002 is Vicente Fox, the singular terms ‘current president of Mexico in August 2002’ and ‘Vicente Fox’ are interchangeable *salva veritate*. In consequence, substituting ‘current president of Mexico in August 2002’ for ‘Vicente Fox” in the logical truth ‘The current president of Mexico in August 2002 is the current president of Mexico in August 2002’ results in the synthetic truth ‘The current president of Mexico in August 2002 is Vicente Fox’. An endorser of this account may object that distinguishing between ‘current president of Mexico in August 2002’ and ‘Vicente Fox’ remains possible. The terms cannot substitute for each other in a sentence like ‘Necessarily the current president of Mexico in August 2002 is the current president of Mexico in August 2002’, because ‘Necessarily Vicente Fox is the current president of Mexico in August 2002’ is false. However, Quine retorts, this objection begs the question.

> The above argument supposes we are working with a language rich enough to contain the adverb “necessarily’, this adverb being so construed as to yield truth when and only when applied to an analytic statement. But, can we condone a

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[20]. (Quine 1951a, 28)

[21]. Quine distinguishes cognitive analyticity from “synonymy in the sense of complete identity in psychological associations or poetic quality.” *Ibidem*. 28

[22]. *Ibidem* 28

> language which contains such an adverb? Does the adverb really make sense? To suppose that it does is to suppose that we have already made satisfactory sense of ‘analytic’. Then what are we so hard at work on right now?[23]

It is clear that Wittgenstein’s grammatical method is very similar to that of Section III in “Two Dogmas”. However, they are also significantly different, and these differences are strong enough to elude Quine’s criticisms. First of all, Wittgenstein’s interchangeability criterion is not *salva veritate*, but *salva grammaticality*. Second, it is not an attempt at defining general synonymy, but grammatical synonymy. In other words, it applies only to terms of grammar, not to all terms in general. Hence, it does not attempt to reduce genuine semantics to syntax – certainly a doomed enterprise. It attempts to give a synonymy criteria for those words whose grammar entirely determines their meaning.

Wittgenstein’s distinction between genuine propositions and those of grammar is similar to that between analytic and synthetic statements. However, Wittgenstein’s distinction presumes nothing about its empirical nature, while Quine’s primary concern is with the empirical dimension of the analytic/synthetic distinction. Wittgenstein’s distinction between genuine propositions and those of grammar is closer to the current logical distinction between syntax and semantics. Wittgenstein bases his distinction at the level of propositions on a distinction at the level of concepts and objects. For Wittgenstein, terms of grammar are those whose grammar entirely determines their meaning. Since grammatical concepts lack intensionality, co-extensionality offers suitable synonymity criteria for them.

Indeed, Wittgenstein never maintained that grammar fully determined the meaning of every expression in a language. However, he argued that it did for those he called ‘of grammar’. In Wittgenstein’s grammar, two terms are grammatically

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[23]. *Ibidem* 29

equivalent if they are interchangeable *salva grammaticality* in all contexts. If the terms belong to grammar, they are also synonymous.

At the level of statements, a statement belongs to grammar if its concepts belong to grammar 's vocabulary. In consequence, its grammar completely determines its 'meaning' and 'truth'. Nevertheless, grammar cannot fully determine the truth of other statements, especially those expressing genuine propositions, i.e. possible states of affairs. It can only determine its grammaticality, that is, whether they are well or ill formed. However, since grammaticality is a necessary condition for the expression of possible states of affairs, modality is built into the language's grammar. In consequence, Wittgenstein's grammatical account does not require a previous understanding of analyticity to explain necessity and, hence, is not circular in Quine's sense.

2. Convention and Justification

> But still there was no truth by convention, because there was no truth. (Quine 1963, 392)

The breadth of Quine's arguments in "Truth by Convention" focuses on the foundational role of linguistic conventions. In consequence, it is mostly irrelevant for Wittgenstein's grammatical project, for Wittgenstein clearly found such a foundational enterprise absurd. Hence, his philosophy of mathematics during the middle period is not a conventionalism in that sense.

The target of Quine's anti-conventionalist arguments is linguistic conventions' inability to found mathematics or calculus. In "Truth by Convention," Quine questions linguistic conventions' capacity to justify mathematical or logical truths. However, Wittgenstein's grammatical account of mathematics is not a foundational enterprise. In Wittgenstein's account, rules of grammar certainly have no justificatory power. Wittgenstein most likely would sympathize with Quine's efforts to demonstrate the impossibility of justifying logical and mathematical truths by inferring them from syntactic conventions.

> . . . the difficulty is that if logic is to proceed *mediately* from conventions, logic is needed for inferring logic from the conventions.[24]

Dummett reiterates this point when he says, in his "Wittgenstein on Necessity":

> The moderate conventionalist view was never a solution to the problem of logical necessity at all, because, by invoking the notion of consequence, it appealed to what it ought to have been explaining: that is why it appears to call for a meta-necessity beyond the necessity it purported to account for. The conventionalists were led astray by the example of the founders of modern logic into concentrating on the notion of logical or analytic *truth*, whereas precisely what they needed to fasten on was that of deductive *consequence*. . .[25]

Wittgenstein would agree with Quine and Dummett that logical truths and linguistic conventions do not logically entail each other. If conventions logically entailed logical truths, justifying this relation would itself require logic. 'Logical entailment' and 'justification' are concepts that do not apply to propositions of grammar, at least not in the same sense as they apply to genuine propositions.

If 'to justify *p*' means to demonstrate the truth of *p*, then justification applies only to genuine propositions. Correct calculations are also called mathematical truths, but mathematical truth is not a sub-species of 'truth' in general. For Quine, "We may mark out the intended scope of the term 'logical truth', within that of the broader term 'truth'."[26] However for Wittgenstein, and at least since the *Tractatus*,

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[24]. W. V. O. Quine, *Mathematical Logic* (Cambridge: Harvard University Press, 1951b) 97.
[25]. Michael Dummett, *The Seas of Language* (Oxford: Clarendon Press, 1993) 460.
[26]. (Quine 1963, 386)

the scopes of 'truth' and 'logical truth' do not overlap. In Ramsey's words, "It is important to see that tautologies are not simply true propositions, though for many purposes they can be treated as true propositions."[27] Ramsey presented Wittgenstein's position very clearly in his 'Foundations of Mathematics' where he wrote:

> The assimilation of tautologies and contradictions with true and false propositions respectively results from the fact that tautologies and contradictions can be taken as arguments to truth-functions just like ordinary propositions, and for determining the truth or falsity of the truth-function, tautologies and contradictions among its arguments must be counted as true and false respectively. Thus, if '*t*' be a tautology, '*c*' a contradiction, '*t* and *p*', 'If *t*, then *p*', '*c* or *p*' are the same as '*p*', and '*t* or *p*', 'if *c*, then *p*' are tautologies.[28]

For Wittgenstein, the 'being true' predicate has only an evaluative function when applied to tautologies. Hence, it means something different when applied to genuine propositions. In the logical calculus of propositions, being true is nothing more than having 'truth' as its truth-value. In the truly semantic case, being true means that the proposition is the case. "For what does a proposition's '*being true*' mean? '*p*' *is true* = *p*. (That is the answer.)"[29] In the case of tautologies and contradictions, nothing could or could not be the case. In consequence, saying that they are true (or false for that matter) in the same sense as true genuine propositions makes no sense. The 'being true' predicate defined for genuine propositions does not apply to tautologies or contradictions.

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[27]. Frank P. Ramsey, 'The Foundations of Mathematics (1925)' in *Philosophical Papers* (Cambridge: Cambridge University Press, 1990) 173.
[28]. *Ibidem* 174.
[29]. *Remarks on the Foundations of Mathematics* Pt. I, Appendix I, §6