What does ‘&’ mean?

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I. In the ongoing debate on the meaning of logical connectives[1], two families of theories have surfaced as main contenders.[2] On the one hand, *representational* semantics hold that the meaning of a logical connective is determined by its contribution to the truth conditions of propositions containing it. On the other hand, *inferentialist* theories (sometimes called ‘functional’ or ‘conceptual’ role semantic theories as well)[3] claim that “the meanings of logical constants are determined by certain characteristic implications.”[4] Most commonly, these characteristic implications correspond to the ‘introduction’ and ‘elimination’ rules of a natural deduction system. For inferentialists, therefore, the meaning of a logical connective is

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[1] . There is a widespread ambiguity regarding the term ‘logical connectives’. For the purposes of this paper, I will use the term to make reference to constant symbols in the language of our logical calculi, like ‘&’, ‘v”, etc. Other authors prefer to restrict their use of the term for the meanings of such symbols.

[2]. I borrow the distinction from Robert B. Brandom *Articulating Reasons. An Introduction to Inferentialism* (Cambridge: Harvard University Press 2000) 45

[3]. Block (2000)

[4]. Gilbert Harman, ‘The Meaning of Logical Constants” *Truth and Interpretation: Perspectives on the Philosophy of Donald Davidson*, edited by Ernest Lepore (Oxford, Blackwell: 1986) 126

closely connected to its deductive calculus.[5] For representationalists, on the other hand, a logical connective's meaning is more intimately connected to its interpretation. Traditionally, the interpretation of a formal language is taken to "establish the link between the formulas and what they stand for".[6] This is the reason why the inferential account is sometimes called '*syntactic'*, while the representational one is called '*semantic*'.

The first approach originates in the work of Peirce, and is implicit in the work of Russell, Ramsey and others. However, it did not become of age until the seminal work of Tarski on the notion of 'truth' for artificial languages.[7] The second one can be traced as far back as Frege's original logical calculus, the *Begriffsschrift* and finds inspiration in Wittgenstein's passages like the following:

> The rules of logical inference cannot be either wrong or right. They determine the meaning of the signs.[8]
>
> We can conceive the rules of inference –I want to say– as giving the signs

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[5]. It is important to notice that . . . "To begin with, inferential role semantics is the doctrine that meaning is inferential role. . . In the first instance, it is sentences, not words, that one infers from and to, and thus it is sentences that *have* inferential roles. Nonetheless, we can think of the inferential role of a word as represented by the set of inferential roles of sentences in which it appears. The notion of the inferential role of a word (and likewise for other sub-sentential constituents) is a made-up idea, but this is an obvious way to make it up." Ned Block, "Holism, Hyper-analyticity and Hyper-compositionality" *Mind & Language* **8**, 1, 1993. 1,2 "Let's call the totality of the inferences to which a sentence is capable of contributing, its *total inferential role*. A sub-sentential constitutent's total inferential role can then be defined accordingly, as consisting in the contribution it makes to the total inferential role of the sentences in which it appears. [¶] Against this rough and ready background, an *inferential role semantics* is just the view that there is some construct out of an expression's total inference role that constitutes its meaning what it does. Let us call this construct an expression's *meaning-constituting inferential role*, or MIR for short." Paul A. Boghossian, "Does an Inferential Role Semantics Rest Upon a Mistake?" *Mind & Language*, **VIII**, 1 (1993) 27.

[6]. Jaroslav Peregrin, "Interpreting Formal Logic" *Erkenntnis* 40 (1994) 5

[7]. For a critical survey of the development of the representational account out of Tarski's theory and its relation to the inferentialist position, cf. Hartry Field "Tarski's Theory of Truth' *The Journal of Philosophy*, **LXIX**, 13 (1972) 347-375

[8]. Ludwif Wittgenstein, *Remarks on the Foundations of Mathematics* (Cambridge, MIT Press: 1956) 24

> their meaning, because they are rules for the use of these signs.[9]

However, it finds its cornerstone in a much-quoted passage from Gerhardt Gentzen's "Investigations into Logical Deduction."[10] Regarding the derivation rules of his sequent calculus, he wrote that

> The introductions represent, as it were, the "definitions" of the symbols concerned, and the eliminations are no more, in the final analysis, than the consequences of these definitions.[11]

Lately, this line of thought has been pursued, in some way or another, by logicians and philosophers like Sellars, Gilbert Harman, Michael Dummett, Ned Block, Robert Brandom and Jaroslav Peregrin.

To better understand the proposed two ways of determining the meaning of a logical connective, let us take '&' as an example. From a representationalist point of view, the meaning of this logical constant is given in its truth table:

p	*q*	*p&q*
T	T	T
T	F	F
F	T	F
F	F	F

This table must be read as saying that if *p* is true and *q* is true, then *p&q* is also true; if *p* is true and *q* is false, *p&q* is false as well; and so on.

For the inferentialist, in contrast, the meaning of this logical connective is determined by its introduction and elimination rules. Traditionally, these are the

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[9] Ludwif Wittgenstein, *Remarks on the Foundations of Mathematics* (Cambridge, MIT Press: 1956) 398

[10]. Cf. Francis J. Pelletier, "A Brief History of Natural Deduction" *History and Philosophy of Logic*, 20 (1999) 1-31

[11]. *American Philosophical* Quarterly, Vol. I, No.4, Octubre 1964: 295, §5.13

following:

Conjunction Introduction:

$$\frac{\begin{array}{c} p \\ q \end{array}}{p\&q}$$

Conjunction Elimination:

$$\frac{p\&q}{p}$$

$$\frac{p\&q}{q}$$

It is a widespread opinion that, at least for truth functional connectives, both approaches can be easily shown to be equivalent.[12] This is achieved by defining logical inference (the key concept in inferentialists theories) in terms of truth conditions (the key concept in representational theories). Accordingly, to say that one proposition logically implies another is to say that the first cannot be true without the other being true. Under this definition of logical inference, introduction and elimination rules provide the same information about the meaning of logical connectives as their truth table. In the case of conjunction, for example, introduction and elimination rules would say that, necessarily

(Introduction) If *p* and *q* are true, then *p&q* is true as well

(Elimination) *p&q* is true only if both *p* and *q* are true

Taken together, they would say that *p&q* is true if, and only if *p* and *q* are both true, which is exactly what the truth table for conjunction tells us.

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[12]. In Jaroslav Peregrin's words, that "the usual [truth-table] meanings we ascribe to expressions [can] be construed as 'disguised' inferential roles." Jaroslav Peregrin, "Meaning as Inferential Role" Logica Yearbook 2002, *forthcoming*.

I call this the 'collapse argument.' It plays an essential role in the current debate between inferrentialists and representationalists, regularly used on both sides to argue for a 'reduction' of one type of semantics to the other

Gilbert Harman (1986) has shown that this argument does not work so well for the rest of the truth-functional connectives, as it does with conjunctions.[13] However, I want to show that even for the apparently unproblematic case of conjunction, the transformation from one definition to the other is not so straightforward either. Mostly, I want to argue that no true inferentialist should be convinced by the collapse argument.

II. There are different ways of rejecting the collapse argument. The first and obvious one would be to reject the truth-conditional definition of logical implication. I have no space here to pursue that line of argumentation. Instead, I will criticize the way the collapse argument reads the elimination and implication rules. I will sustain that, even before applying the truth-theoretical definition of logical implication, the collapse argument requires a non-inferentialist reading of the introduction and elimination rules, thus making it unacceptable for a true inferentialist.

Let me start by giving a brief account of what exactly is an inferentialist theory of meaning, when applied to logical formalism. Inferentialists are so called,

[13]. It is important to tell that the aim of Harman's argument in (1986) is not to overcome inferentialism (after all, his non-solipsistic conceptual role semantics is an inferentialism), but only its commitment to characteristic implications expressed by inference rules. Among the 'final remarks' of his paper, he writes: "Even if the meanings of logical constants are determined by their role in inference, it is imprecise to say that these roles are determined by characteristic implications." P. 134

because they advocate a thoroughly inferential account of meaning. This dictum acquires special relevance when applied to the interpretation of logical formalism.[14]

> Now, if logical concepts are entirely fixed by their functions in reasoning, a concept *C* expresses logical conjunction if it serves to combine two thoughts *P* and *Q* to form a third thought *C*(*P*, *Q*), where the role of *C* can be characterized in terms of the principles of 'conjunction introduction' and 'conjunction elmination'.[15]

For an inferentialist, therefore, logical rules and formulae must be read exclusively in terms of inferential operations. Inferentialists diverge on whether these inferential operations are determined by psychological processes or by proof constructions. In the remaining of this paper I will adopt a proof-theoretical vocabulary, but nothing of what I will say next could not be easily expressed in psychological terms as well.

Under an inferentialist theory of meaning, therefore, logical rules must be read as saying something about the validity of proofs, while formulae must be interpreted in terms of allowed proof constructions in the calculus. A rule that says that a formula *A* follows from another formula *B* expresses the logical validity of any proof that has *A* among its premises and *B* as its conclusion.

Now, let us take a new look at the introduction and elimination rules for conjunction:

(Introduction) *p&q* follows from *p* and *q*

(Elimination) *p* and *q* follow from *p&q*

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[14]. According to Brandom, this new "way of thinking about semantics" is also the "basis" of "a new way of thinking about logic" (Brandom (2000) 45). Hence, the inferentialist/representationalist controversy diverges into a parallel philosophical debate regarding the nature of logic: the question whether Logics's central object of study is logical consequence or logical truth. Inferentialist's advocacy of an inferentialist interpretation of logical formalism seems to commit them to place logical truth at a subordinate position in relation to logical consequence.

[15]. Gilbert Harman, "(Non Solipsistic) Conceptual Role Semantics" in E. Lepore (ed.) *New Directions in Semantics* (London: Academic Press, 1987) 137

So read, these rules seem to establish a kind of logical equivalence between the logical connective '&' and the logical word 'and'. This is one of the reasons why some logicians believe that the logical connective '&' is synonymous or is the *formal counterpart* of logical word 'and' in natural language.[16] However, it is important to notice that the word 'and' that occurs in the enunciation of these rules belongs to the meta-language of logic. Under an inferentialist perspective, this would be the language logicians use to talk about the construction of proofs in a formal system. To make this proof-theoretical dimension more obvious we may want to rephrase these readings more explicitly as:

(Introduction) Any proof that includes **both** *p* **and** *q* among its premises, and *p&q* as its conclusion is logically valid.

(Elimination) Any argument that includes *p&q* among its premises, and **either** *p* **or** *q* as its conclusion is logically valid.

Notice that by reading the rules this way, not only the second 'and' (as well as the putative synonymy between '&' and 'and') disappears, but also the apparent topical homogeneity between both rules. On the inferentialist reading, each rule connects the '&' connective with a different sort of proof construction. The introduction rule connects it with the addition of premises to an argument, while the elimination rule connects it with the operation of obtaining different conclusions from the same set of premises. Hence, according to the introduction rule, the connective '&' expresses the conjunction of premises, while the elimination rule defines the connective as expressing the conjunction of conclusions. From a proof-theoretical point of view it is clear that these two conjunctions are different.

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[16]. Cf. Raúl Orayén *Lógica, Significado y Ontología* (México D.F.: UNAM/IIF, 1989) 172-174

Finally, even if we accept the truth-theoretical account of logical inference, we are left not with one, but two truth tables, one for each rule and logical operation. Under the truth-conditional definition of logical inference, the introduction rule for conjunction would determine the following truth table:

p	*q*	*p&q*
T	T	T
T	F	?
F	T	?
F	F	?

while the elimination rule would determine the following one

p&q	*p*	*q*
T	T	T
F	?	?

In other words, even under a truth-conditional reading, the introduction rule does not tell us anything about the truth value of the conjunction **as a conclusion** when the *premises* are not both true. Likewise, the elimination rule does not tell us anything about (the truth value of) the *conclusion* when the conjunction **as a premise** is not true.

The proponent of the collapse argument still needs to establish the correspondence between these two incomplete tables and the single and complete truth table proposed by representationalists. Thus, she is faced with a dilemma: either take one of the incomplete tables and 'fill in the blanks' to obtain a complete one, or integrate both partial truth tables into a complete one. The first horn of the dilemma would be accomplished, for example, by saying that the introduction and/or elimination rule for the connective establishes both the necessary and sufficient conditions for the validity of certain proofs. However, this approach seems to have the inconvenience of reading these inference rules as establishing logical equivalence, instead of logical inference. Hence, it would seem logically inadmissible. However, some version of it has been recently advocated by M. E. Kalderon (2001) and J. Peregrin (*forthcoming*).[17]

Peregrin argues for what he calls an *Exhaustive Assumption (EA)* when translating inference rules into truth tables*:*

> If we manage to turn all the rows ending with T into inferential conditions, we can complete the specification by stimulating minimality: "any other statement fulfilling all the conditions must be inferable from $O(S_1, \ldots S_n)$

In other words, for Peregrin, the information provided by the truth table is not contained in any of the introduction or elimination rules, but in the fact that they are *the* introduction and *the* elimination rules for the aforementioned connective. In consequence, introduction rules must be read under the *minimality*[18] assumption that the inferences they determine are the only way to introduce the connective. Elimination rules, conversely, must also be read under the *minimality* assumption that the inferences they determine are the only ones that allow for the elimination of the connective.

In the case of conjunction, this would mean that the introduction rule establishes the only pattern of inference available for the introduction of the

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[17]. I say 'some version of it', because they do not use this strategy to join the information from the elimination and introduction rules or to complete the truth table of conjunction. They do not consider this strategy to be necessary in the case of conjunction.

[18]. Peregrin's exhaustivity assumption includes a *maximality* principle as well, for completing partial truth tables that include only lines finishing in 'F'. I find his account of those cases as extremely problematic, but at least does not apply to conjunction, so is out of the scope of this paper.

conjunction symbol. In other words, that no other combination of *p* and *q* would entail *p&q*. This way, Peregrin can complete the truth table obtained from the introduction rule as follows:

p	*q*	*p&q*
T	T	T
T	F	**F**
F	T	**F**
F	F	**F**

However, this strategy does not work as nicely for the elimination rule.

On the other hand, taking the second horn of the dilemma would require a philosophical account that brings together both logical operations under a single notion of conjunction. However, we have no logical or philosophical reason to say that joining different premises together in an argument is the same (or inverse) operation than obtaining different conclusions from the same argument.

In either case, the main hurdle to overcome would be to give a philosophical account of the ambiguity emerging from the inferentialist account. One must explain how two apparently different logical constructions (adding new premises and obtaining new conclusions) may determine one single logical concept (conjunction). This may be accomplished by explaining either which *one* of the two aforementioned conjunctions is *the* logical conjunction, or how both operations of conjunction form a single logical unity. However, it is clear that such account does not yet exist, and that the issue has not been properly raised in the current debate. Consequently, it is correct to say that there is no straightforward reconciliation for the inferentialist and representationist theories of meaning for logical connectives.

III. Finally, however, one may still wonder why would we need to conciliate both approaches after all. There is still the alternative of proposing a two-factor theory of meaning for logical connectives. On this vision, the meaning of a logical connective is a composite of two sorts of semantic contents: its *conceptual content,* determined inferentially, and its representational content, determined by its truth table.

> On this view, meaning consist of an internal, "narrow" aspect of meaning . . . and an external referential/truth-theoretic aspect of meaning . . .[19]

Besides the usual problems burdening two-factor theories of meanings,[20] this strategy is specially difficult to maintain in the case logical connectives, because logical terms have traditionally been defined precisely as those elements of language whose meaning is entirely inferential. As such, they are supposed to have no representational role besides that relevant for inference and logical implication. A representational account of meaning, in the case of logical terms, cannot go beyond establishing its inferential role. Logical terms differ from other non-logical terms in their having only conceptual content. This was the original definition of Frege in his *Begriffsschrift* (1879), and survives in current logical orthodoxy. The relevant passage from Frege reads as follows:

> There are two ways in which the content of two judgements may differ; it may, or it may not, be the case that all inferences that can be drawn from the first judgement when combined with certain other ones can always also be drawn from the second when combined with certain other ones can

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[19]. Ned Block, "Conceptual Role Semantics" in Edward Craig (ed.) *The Routledge Encyclopedia of Philosophy* (London: Routledge, 2000)

[20]. For a summary account of the difficulties of two-factor semantics, see Block (2000)

> always also be drawn from the second when combined with the same other judgements. The two propositions 'the Greeks defeated the Persians at Plataca' and 'the Persians were defeated by the Greeks at Plataca' differ in the former way; even if a slight difference of sense is discernible, the agreement in sense is preponderant. Now I call that part of the content that is the same in both the conceptual content. Only this has significance for our symbolic language [*Begriffsschrift*] . . . In my formalized language [*Begriffsschrift*] . . . only that part of judgements which affects the possible inferences is taken into consideration. Whatever is needed for a correct inference is fully expressed; what is not needed is . . . not. (Frege 1879 §3)

This conception was the original motivation behind inferentialism, and admiting a two-tier theory of meaning would go directly against it. Hence, it is inconsistent with the inferentialist position to hold a two-tier account of meaning, at least *for logical connectives*.

In the end, the inferentialist must bite the bullet and reject the representationalist's truth-functional story both as an equivalent and/or as a supplementary account of the meaning of logical connectives.

APPENDIX: What the Collapse Argument is not.

The collapse argument is not an argument for the reduction of the inferrential content of a connective to its non-referrential.

In other words, truth tables themselves can be read both inferrentially and representationally. For representationalists, each line of the truth table establishes the truth-conditions of a complex sentence in terms of the truth conditions of its components sentences. For inferrentialist, a truth table keeps expressing inferrential connections, and each line must be read as establishing a valid inference form the turth or falsity of the component sentences to the truth or falsity of the component statement. In the case of conjunction notice that when we say that *p&q* is true only if

p and q are both true, we are using both the conjunction and implication notions as marked by the occurrence of the so-called logical words "and" and "only if". In other words, truth tables cannot be interpreted as defining the meaning of logical connectives in non-inferrential terms.

Furthermore, this jump from direct inferences (from p and q to $p\&q$) to truth-conditional inferences (from the truth of p and the truth of q to the truth of $p\&q$) has its (very high) price. If it is not $p\&q$ which follows from p and q, but **the truth of** $p\&q$ which follows from **the truth of** p and **the truth of** q,

Hence, instead of three patterns of inference, the truth table establishes four patterns of inference:

p is true
q is true

$p\&q$ is true

p is true
q is false

$p\&q$ is false

p is false
q is true

$p\&q$ is false

p is false
q is false

$p\&q$ is false

However, if these four basic patterns of inference establish the meaning of the connective, then we must then read them as

It is true that p is true
It is true that q is true

It is true that $p\&q$ is true

It is true that *p* is true
It is false that *q* is true

It is true that *p&q* is true

It is false that *p* is true
It is true that *q* is true

It is true that *p&q* is true

It is false that *p* is true
It is false that *q* is true

It is true that *p&q* is true

and

It is true that *p* is true
It is true that *q* is false

It is true that *p&q* is false

It is true that *p* is true
It is false that *q* is false

It is false that *p&q* is false

It is false that *p* is true
It is true that *q* is false

It is false that *p&q* is false

It is false that *p* is true
It is false that *q* is false

It is false that *p&q* is false

then what happens when *p* or *q* are not actually true. In the direct inferrentialist case, it is still possible for *p&q* to follow from *p* and *q*, even when one of them is false. However, in the truth table case, those lines in the table which contradict reality must be read subjunctively. Hence, they would have to say that if *p* and *q* **were** true, *p&q* would also be true.