

Mathematics as Grammar

'Grammar' in Wittgenstein's
Philosophy of Mathematics
during the Middle Period

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May this humble work
serve to glorify the Lord.

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This dissertation was prepared in accordance with the latest version of Indiana University's *Guide to the Preparation of Thesis and Dissertation* by the Research and the University Graduate School, and the guidelines of the University Graduate School *Bulletin*. It also follows *The Chicago Manual of Style* and Kate L. Turabian's *Manual for Writers of Term Papers, Theses, and Dissertations*. As recommended by the University Graduate School, *Webster's New Collegiate Dictionary* served as authority on English spelling and usage. For German, *Cassell's German Dictionary* by Karl Breul served this function.

1. Furthermore, her grammatical and stylistic corrections served as informal evidence to the main hypothesis of this dissertation. Her corrections included not only obviously grammatical rules, but also transformation rules with a natural logical interpretation, such as DeMorgan rules or exchange of quantifiers.

Abstract

This dissertation looks to make sense of the role ‘grammar’ plays in Wittgenstein's philosophy of mathematics during the middle period of his career. It constructs a formal model of Wittgenstein's notion of *grammar* as expressed in his writings of the early thirties, addresses the appropriateness of that model and then employs it to test Wittgenstein's claim that mathematical propositions are ultimately grammatical.

In order to test Wittgenstein's claim that mathematical propositions are grammatical, the dissertation provides a formalized theory of grammatical analysis and applies it to a portion of language big enough to contain numerical expressions. It attempts to prove that, if the object language contains the appropriate numerical expressions, the resulting grammar will include at least some rules with a natural mathematical interpretation. In particular, it tries to show that Wittgenstein's grammatical analysis of the ordinary use of numerical expressions yields familiar theorems of arithmetic.

The dissertation also endeavors to extend these results to a coherent picture of Wittgenstein's peculiar view of mathematics as grammar. It fits the formal results into the larger picture of Wittgenstein's philosophy of mathematics during this period. A large portion of the dissertation explains how, by combining the notions of grammar and mathematics, Wittgenstein allowed himself to offer original answers to some central questions in the philosophy of mathematics. In this regard, the dissertation pays special

attention to four main issues: (i) Wittgenstein's use of the term 'grammar' in his philosophical writings of this period, in comparison with traditional understandings of 'grammar', (ii) mathematics as part of the syntax of language, (iii) his explanation of mathematical application, and (iv) Wittgenstein's account of mathematical necessity. Taking these points in consideration situates the dissertation's results in the larger philosophical discussion of these themes.