Chapter 1
Introduction

In the early 1930s, Ludwig Wittgenstein advanced the thesis that mathematical statements are ultimately grammatical. Most of Wittgenstein's audience at the time were philosophers and mathematicians with little or no knowledge of linguistics. They found the idea that mathematics could be part of grammar – what they called "the dullest of school subjects" – inconceivable. Consequently, they interpreted 'grammar' as some esoteric logical syntax of scientificlanguage.[1] Despite the profound development of grammatical studies in the 20th century, many philosopher's prevalent attitude towards grammar has changed little. The time has come for a theoretical reevaluation of Wittgenstein's thesis in light of the most recent developments in theoretical grammar. This dissertation explains grammar's role in Wittgenstein's philosophy of mathematics during the early thirties. It answers two central questions, 'Can mathematical propositions be grammatical?' and 'How does Wittgenstein support this thesis?' It explains Wittgenstein's claim that mathematical propositions are ultimately grammatical and describes this claim's role in his philosophy of mathematics during that period.

1. In his "Intellectual Autobiography," Carnap writes about the Vienna Circle, "When we found in Wittgenstein's book [The *Tractatus Logico Philosophicus*] statements about "the language," we interpreted them as referring to an ideal language; and this meant for us a formalized symbolic language. Later Wittgenstein explicitly rejected this view." Paul Arthur Schilpp, ed. *The Philosophy of Rudolf Carnap*, The Library of Living Philosophers, vol. XI (La Salle: Open Court, 1963) 29.

Philosophers have questioned whether or not mathematics is part of the formal grammar of language for more than a century. This question lies at the center of the debate between Carnap and Bar-Hillel, on the one side, and Gödel, Tarski and Quine on the other. Carnap asserted that his philosophy of mathematics as syntax originated in Wittgenstein. In consequence, it is important to clarify whether or not Wittgenstein held a view similar to the one Carnap championed and, if so, to defend him against Quine's criticisms.

Throughout the *Big Typescript*, *Philosophical Grammar* and *Philosophical Remarks,* Wittgenstein developed most of his ideas on the philosophy of mathematics through examples from elementary arithmetic. Following his own presentation, this dissertation concentrates on the case of mathematical numerical expressions and their calculi. However, since Wittgenstein's notion of calculation includes mathematical processes like drawing geometrical figures, and proving theorems within a formal system, his results rightfully extend over all mathematics. On this topic, S. G. Shanker writes at the beginning of *Wittgenstein and the Turning-Point in the Philosophy of Mathematics*,

> The more [Wittgenstein] addressed the fundamental confusions underlying the 'foundations crisis' the more strongly he began to feel that the philosophical problems which surface in the various realms of higher mathematics are merely more complex versions of the same issues which arise in elementary arithmetic. For example, the type of problems that emerged with the construction of the transfinite cardinals are essentially the same as those that characterize the construction of any new number system. Hence, Wittgenstein sought to gain in perspicuity what he lost in detailed application by presenting his criticisms of the questions which prefigure in higher mathematics in the context of the problems which occur in elementary arithmetic.[2]

The first chapter develops some preliminary notions in Wittgenstein's philosophy of mathematics. It introduces the notions of grammatical concept and object. Wittgenstein bases his argument on these two notions.

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2. S. G. Shanker, *Wittgenstein and the Turning-Point in the Philosophy of Mathematics* (Albany: State University of New York Press, 1987), 5.

The second chapter develops Wittgenstein's idea that mathematical propositions connect calculations with their final results. For example, the arithmetical proposition '3 + 4 = 7' says that adding three to four results in seven. In developing this idea, Wittgenstein criticizes alternative philosophical approaches to mathematical numerical expressions [*Zahlangaben*] – in particular, Frege's and Ramsey's accounts of arithmetical equations, Frege's seminal work on the concept of number, and Russell's notion of cardinality.

The third chapter explains why mathematical propositions are calculation rules. Calculation is a rule-governed linguistic practice and calculi are grammatical systems. Every mathematical calculus is a linguistic system with its own grammar. This grammar determines correct or incorrect calculations. The calculation's result is correct if the calculation itself is correct, not vice versa. Wittgenstein opposed the Platonists' view of calculations as *expeditions* into uncharted mathematical territory. Instead, he envisions calculations as *searches* over well defined grammatical spaces.

Wittgenstein's claim that mathematical propositions are grammatical also means that they are part of the grammar of natural language. Understanding mathematics' relationship to natural language requires analyzing mathematical application [*Anwendung*]. This analysis takes place in the third chapter.

From the point of view of calculation, pure and applied mathematics are not significantly different. Pure and applied mathematics consist entirely of calculations [*Rechnungen*]. The grammar of mathematical expressions is the same in natural language and in pure mathematics. The calculus determines the grammar of mathematical expressions in natural language as well as in calculation. In consequence, mathematical calculi are part of the grammar of natural language. Mathematics are applicable only inside some defined calculus [*Kalkül*]. Calculation says nothing about matters outside the calculus. Mathematical

calculations help solve non-mathematical problems. They provide the grammar of non-mathematical hypothesis. However, they do not justify or entail the hypothesis' truth.

The fourth chapter explains Wittgenstein's use of 'grammar'. It demonstrates formally that numerical calculi form grammatical systems. It also provides a formalized theory of grammatical analysis. The fifth chapter applies this theory to prove that mathematical calculi are part of natural language grammar. They are part of the grammar of the segment of natural language in which they occur. If the object language contains the appropriate mathematical expressions, the resulting grammar includes at least some rules with a natural mathematical interpretation. In particular, the grammatical analysis of numerical expressions in natural language obtains familiar arithmetical axioms.

Using a formal method to study Wittgenstein's philosophy is highly controversial. He explicitly opposed formal methods in philosophy.[3] However, good reasons support adopting a formal method. Working with the notion *grammatical* in a formal context is essential to clarify Wittgenstein's claim that mathematical propositions are grammatical. The formal approach provides a rigorous understanding of the adjective 'grammatical'. However, the ultimate subject of this formal reconstruction and analysis is Wittgenstein's philosophy of mathematics. Even though some of this analysis' formal results might have importance of their own, formal logic plays only a supporting role.

Finally, the seventh chapter addresses the Carnap-Gödel debate on the syntactic nature of mathematics. It elaborates on notions developed in previous chapters to explain the analytic nature of mathematical propositions. It develops Wittgenstein's account of syntactic necessity, and defends it from Quine's arguments.

3. "He [Wittgenstein] had a skeptical and sometimes even a negative view of the importance of a symbolic language for the clarification and correction of the confusions in ordinary language. . ." (Carnap 1963, 29)

I. Wittgenstein's Philosophy of Mathematics during the Middle Period

A. Wittgenstein

Wittgenstein's philosophical ideas, not to mention his unconventional means for expressing them, are so radical that interpreters cannot arrive at a consensus on how to approach his work or decide upon its lasting significance. He figures prominently among the early masters of so-called analytical philosophy, like Frege, Russell, the Vienna circle and the Oxford school of ordinary language philosophy. His name frequently appears in connection with thinkers as diverse as Shopenhauer, Kirkegaard, Heidegger or Derrida. Authors allude to his work even in topics like gardening, rhetoric, mysticism, architecture, and deep psychology.

The complex historical conditions of his life and thought make him an eminent figure in the intellectual history of the 20th Century. He was fortunate enough to come of age in *fin-de-siécle* Vienna, at the same time than such central figures of early 20th Century Western History as Gustav Mahler, Sigmund Freud, Gustav Klimt and Adolf Loos. A quarter of a century later, he partook in another intellectual revolution, when the Vienna Circle drew him into its deliberations. His name is just as much connected with the intellectual history of his hometown as it is with that of Cambridge. Upon his first arrival in Cambridge, Wittgenstein found himself surrounded with some of the leading English intellectuals of the period: Bertrand Russell, Alfred North Whitehead, John Maynard Keynes and Lytton Strachey. During the thirties and early forties, he again became part of a strong academic community featuring G. E. Moore, and Pierro Straffa. As a professor, Wittgenstein also enjoyed a following of students like Norman Malcolm, Rush Rhees and Elizabeth Anscombe, who helped spread his ideas throughout the English-speaking world.

Despite the continuous effort of some interpreters to frame his iconoclastic thinking within a philosophical tradition or doctrine, Wittgenstein did not even adhere to his own

doctrine. In *A Biographical Sketch*, composed not long after Wittgenstein's death, G. H. von Wright wrote:

> Wittgenstein [partly] repudiated the results of his own influence. He did not participate in the worldwide discussion to which his work and thought had given rise. He was of the opinion --justified I believe-- that his ideas were usually misunderstood and distorted even by those who professed to be his disciples. He doubted that he would be better understood in the future. He once said that he felt as though he were writing for people who would think in a quite different way, breathe a different air of life, from that of present-day men.[4]

Wittgenstein was an extreme example of a Mexican *'mamón'*. His intense personality elicited extreme responses in the few who met him in person. People admired and feared him. Most of all, his extraordinary intelligence and aloofness charmed them. When asked to write an assessment of Wittgenstein for a symposium, Norman Malcolm wrote in 1960:

> Wittgenstein's conversation made an overwhelming impression because of the united seriousness and vivacity of his ideas, and also because of the expressive mobility of his beautiful face, the piercing eyes and commanding glance, the energetic movements and gestures. In comparison, someone has remarked, other people seemed only half alive.[5]

He refused to associate with those he found undesirable. Some say that Wittgenstein avoided making acquaintances, but needed and sought friendships.[6] According to Jaakko Hintikka, Wittgenstein "was a philosophical genius, but he was socially and intellectually a lone wolf who did not assume any responsibility for, or even exhibit an interest in, many of the institutions of our society and culture."[7] Although he tried to avoid publicity, his reclusive behavior prompted the growth and dissemination of numerous legends about his perso-

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4. G. H. von Wright, "Ludwig Wittgenstein: A Biographical Sketch" in Norman Malcolm ed. *Wittgenstein* (Minneapolis: University of Minnesota, 1960), 15.

5. Ibid.

6. Ibid., 31.

7. Jaakko Hintikka, "Who is about to Kill Analytic Philosophy?" in Anat Biletzki and Anat Matar, *The Story of Analytic Philosophy* (New York: Routledge, 1998), 259.

nality. As a result, his name appears today in detective novels and art films almost as often as in history books and philosophy journals.

In contrast with the pervasive influence of his ideas, Wittgenstein remains an elusive philosopher. Because he resisted theorizing, wrote in an aphoristic style, and expressed his thoughts in a highly personal, even existential tone, Wittgenstein did not fit comfortably in academia.[8] Although he taught in Cambridge for more than a decade, Wittgenstein disdained the scholastic pedantry of the academic philosophy of his days. On the contrary, he continually challenged the protocols of academia both in his writings and professional practice. In a personal description of Wittgenstein, Gilbert Ryle wrote:

> He loathed being connected with academic philosophers, and he avoided academic chores. After 1929 he attended no conferences; he did no reviewing for journals; only once did he attend a philosophical meeting in Oxford; he was inaccessible to visiting philosophers; he read few, if any, of the philosophical books and articles that came out during his last 25 years.[9]

Paradoxically, academia could not have more thoroughly embraced another philosophical figure in the 20th century. Every year, the study of his philosophical ideas fills numerous articles, books, dissertations and journals. Nevertheless, much of his rich and complex work remains unexplored.

8. Some interpreters have argued that a methodical reading of Wittgenstein's texts does not adequately match the disjoint quality of his writing. However, this appraisal of Wittgenstein's thought is unjustifiably condescending.

9. Gilbert Ryle, “The Work of an Influential but Little-known Philosopher of Science: Ludwig Wittgenstein” (Shanker 1986, 138)

B. Wittgenstein and Mathematics

> Die philosophische Klarheit wird auf das Wachstum der Mathematik das gleichen Einfluß haben, wie das Sonnenlicht auf das Wachsen der Kartoffeltriebe. (In dunkeln Keller wachsen sie meterlang.)
>
> PG §25 p.750
>
> Philosophical clarity will have the same effect on the growth of mathematics as sunlight has on the growth of potato shoots. (In a dark cellar they grow meters long.)
>
> PG §25 p. 381

Wittgenstein's philosophical writings on mathematics call into question the boundaries between philosophy and mathematics. His work reveals the philosophical issues behind some problems otherwise considered as purely mathematical. For example, he found that neither logic nor any other calculus could serve as a foundation for arithmetics. For this reason some mathematicians have accused Wittgenstein of technical incompetence. However, when trying to expose Wittgenstein's mathematical errors, these mathematicians have found themselves dealing with deep philosophical issues. In his philosophical writings on mathematics, Wittgenstein did not blur the line between mathematics and philosophy, but challenged the traditional way of separating them.

On entering the subject of mathematics, some authors find it proper to state that Wittgenstein disclaimed any specialist knowledge of mathematics. For example, Jaakko Hintikka writes,

> Wittgenstein had no sympathy for, or real understanding of, mathematical . . . theorizing. For all his aesthetic sensibilities, he had for instance no feeling for the elegance and power of a real mathematical theory. There are no indications that he had any appreciation of, or even knowledge of, such things as Galois theory, the calculus of residues, Gauss-Riemann surface theory, or the theory of Hilbert spaces.[10]

"For Wittgenstein has been accused," remarks S. G. Shanker, "not simply of subversive – even anarchic – tendencies, but even worse, he has repeatedly been charged with that most

[10]. (Hintikka 1998, 259)

heinous of crimes: technical incompetence."[11] However, his knowledge of mathematics did not derive "from extensive reading, but from a working familiarity with mathematical techniques."[12] Elementary arithmetic covers a big portion of Wittgenstein's *Remarks on the Foundations of Mathematics*. However, this is not evidence of his technical incompetence. His manuscripts of the thirties contain meticulous and detailed analyses of a wide range of non-elementary mathematical issues: from Cantorean transfinite number theory and the continuum problem to Skolem's recursive proof and Hilbert's various attempts to construct a consistency proof.[13]

Through the many changes in Wittgenstein's life and thought, few concerns remained of greater importance than the foundations of mathematics. As early as his late teenage years, Wittgenstein had already developed an "amateur's fascination" with mathematics. His interest in the philosophical aspects of mathematics began during his days at Manchester University. After earning his certificate in engineering at the *Technische Hochschule* in Berlin-Charlottenburg, Wittgenstein registered in the fall of 1908 at Manchester as an engineering research student. At this time, discussions with Horace Lamb and lectures from J. E. Littlewood led him to read Russell's recently published *The Principles of Mathematics.* Eventually, his growing interest in mathematical logic and the philosophy of mathematics led him to abandon engineering and to focus on these subjects. After leaving Manchester, Wittgenstein visited Gottlob Frege, who advised him to study logic with Russell at Cambridge University. Once there, Wittgenstein's philosophical vocation extended beyond the limits of logic and mathematics. From that moment on, he became vitally immersed in philosophy of language, metaphysics and, later, mysticism, ethics and aesthetics as well.

11. S. G. Shanker, "Introduction: The Portals of Discovery" (Shanker 1986, 1)

12. (von Wright 1960, 33) Nevertheless, this is an understatement. Wittgenstein kept up with contemporary developments in mathematics through extensive reading.

13. (Shanker 1986, 3)

Wittgenstein's attitude towards these issues was not that of detachment, but vigorous engagement. He found the aloofness of academia increasingly unbearable. So much that, when World War I erupted, he abandoned Cambridge.

According to most accounts, Brower's lecture “Mathematics, Sciene and Language” [*Mathematik, Wissenschaft und Sprache*], which Wittgenstein attended in Vienna in 1928, moved him to resume philosophy. Wittgenstein did not leave the lecture converted to Brouwer's intuitionistic project, but something in that lecture struck a responsive chord in him. As a matter of fact, he focused on the philosophy of mathematics, figuring prominently in his typescripts, manuscripts and lectures from the period. According to P. M. S. Shanker, "Approximately one-third of the *Big Typescript* is concerned with the philosophy of mathematics; indeed, it should not be forgotten that in the years between 1929 and 1944 about half of Wittgenstein's writings were on his subject"[14].

Philosophy of mathematics remained central to Wittgenstein's thought during the later period of his life. A broad selection of his remarks on this topic written in the years 1937-1944 were published in 1956 as *Remarks on the Foundations of Mathematics*. Wittgenstein's originally intended to incorporate these remarks to the *PhilosophicalInvestigations.*[15] In fact, Part I of the *Remarks* was part of the first version of the *Investigations* written in Norway during 1937. After being elected to hold Moore's chair at Cambridge, Wittgenstein delivered a series of lectures on the philosophy of mathematics. These lectures were later published as *Wittgenstein's Lectures on the Foundations of Mathematics*. However, these reflections are not the last chapter in Wittgenstein's life-long liaison with mathematics and its philosophical problems. Almost until his final days, he continued writing on these topics. Rush Rhees remembers that when John Wisdom asked Wittgen-

14. Ibid. 87

15. G. H. von Wright, R. H. Rhees and G. E. M. Anscombe, editor's preface to Ludwig Wittgenstein. *Remarks on the Foundations of Mathemathic* (Cambridge: MIT Press, 1956), vi.

stein in 1944 to suggest a dictionary entry about his philosophy, he wrote just one sentence: "He has concerned himself principally with questions about the foundations of mathematics."[16]

Despite his extreme enthusiasm for mathematics and its philosophy, Wittgenstein eliminated most remarks on the foundations of mathematics from the published version of the *PhilosophicalInvestigations*. In consequence, his ideas on this subject remained largely unknown until the posthumous publication of his manuscripts, typescripts and lecture notes. Wittgenstein's first posthumous volume on mathematics was the *Remarks on the Foundations of Mathematics*, from 1956. Unfortunately, the philosophical community did not react encouragingly. In his introduction to the third volume of *Ludwig Wittgenstein, Critical Assessments*, Stuart Shanker writes about "a storm of calumny . . . raging" at the aftermath of its publication.[17] Morris Engel writes about "the almost unanimous sense of disappointment and disapproval"[18] in early reviews. Even advocates of Wittgenstein's later philosophy dismissed it as erratic and misinformed. For example, the opening paragraph from Michael Dummett's seminal article from 1959, "Wittgenstein's Philosophy of Mathematics," says,

> From time to time Wittgenstein recorded in separate notebooks thoughts that occurred to him about the philosophy of mathematics. His recently published *Remarks on the Foundations of Mathematics* consists of extracts made by the editors from five of these. Neither it nor any of these notebooks was intended by its author as a book. That it cannot be considered, and ought not to be criticized, as such is therefore unsurprising, though disappointing. Many of the thoughts are expressed in a manner which the author recognized as inaccurate or obscure; some passages contradict others; some are quite inconclusive; some raise objections to ideas which Wittgenstein held or had held which are not themselves stated clearly in the volume; other passages again, particularly those on consistency and on Gödel's theorem, are of poor quality or contain definite errors.

16. Michael Nedo, introduction to *Wiener Ausgabe* (New York: Springer-Verlag, 1993), 57.

17. (Shanker 1986, 1)

18. S. Morris Engel, "Wittgenstein's *Foundations* and Its Reception" (Shanker 1986, 146), 257-68.

Dummett also wrote that some of Wittgenstein's remarks are "plainly silly," "extremely hard to swallow," "extraordinarily difficult to take . . . seriously," "thin and unconvincing." Dummett was not alone in his disappointment with the *Remarks on the Foundations of Mathematics*. Georg Kreisel ended his review saying, "It seems to me to be a surprisingly insignificant product of a sparkling mind."[19] Alan Ross Anderson wrote in his review that "it is not hard to reach the conclusion that Wittgenstein failed to understand clearly the problems with which workers in the foundations have been concerned."[20] A page later he wrote, "It is very doubtful whether this application of his method to questions in the foundations of mathematics will contribute substantially to his reputation as a philosopher."[21]

Comments of this sort set the tone for almost all further discussion. It came as no surprise, then, that the rest of Wittgenstein's writings on the subject received a similar condescending and dismissive response. Even today, some Wittgenstein scholars allege that most of his remarks on the philosophy of mathematics are largely wrong, and that having a complete picture of Wittgenstein's thought is the only reason to study them.[22] In consequence, some of Wittgenstein's least studied writings are from the middle period. In his introduction to the 1996 *Cambridge Companion to Wittgenstein*, Hans Sluga admits that Wittgenstein's remarks from the thirties "on the philosophy of mathematics have remained

19. G. Kreisel, "Wittgenstein's *Remarks on the Foundations of Mathemathic*" *British Journal for the Philsoophy of Science* 9 (1958-59) : 158 quoted in. Michael Wrigley, "Wittgenstein's Philosophy of Mathemathics" (Shanker 1986, 183)

20. Alan Ross Anderson, "Mathematics and the Language Game" *The Review of Mathemathic* 11 (1958) : 457 quoted in Morris Engel (Shanker 1986, 147)

21. Ibid. 458.

22. For example, in the preface of his *Wittgenstein on the Foundations of Mathemathic* (Harvard: Cambridge, 1980), Chrispin Wright warns that "the easy stance of eclecticism" – to recover what one thinks is right, and dismiss what one may dislike in Wittgenstein – "is not an option".

among Wittgenstein's most controversial and least explored writings."[23]

C. The Middle Period

Wittgenstein's middle period ranges from his return to Cambridge, early in 1929, to 1933.[24] According to Brian McGuinness and G. H. Von Wright, Wittgenstein visited Cambridge for a holiday, but he quickly decided to stay.[25] Before readmission to Trinity, he stayed with J. M. Keynes at King's College, and later with Lettice and Frank Ramsey. On 18 January 1929, he began as a research student, working towards a Ph.D. degree. However, his status obviously did not correspond to that position. In consequence, the university offered to count his pre-war residence at Cambridge as credit towards the degree and the *Tractatus*, published eight years earlier, as a thesis. Ramsey was formally his supervisor, and Moore and Russell, his examiners. He received his degree on June 18, 1929. During the academic year 1929-30, he lectured on philosophical logic at the invitation of the Moral Sciences Faculty Board. By the end of 1930, he was a fellow of Trinity College. The fellowship extended until the end of the academic year 1935-6, when his Faculty Lectureship also ended.

Wittgenstein initially returned to Cambridge to correct certain difficulties in the

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23. Hans Sluga, introduction to *The Cambridge Companion to Wittgenstein* (New Yok: Cambridge, 1996), 17.

24. In this point, this dissertation sides with Dale Jacquette. In the introduction to *Wittgenstein's Thought in Transition*, he wrote, "I designate [the middle period] from 1929 to 1933. . . The dates are significant and by no means arbitrary. . . In 1930, Wittgenstein began lecturing at Cambridge University. The end of the transition period can be dated approximately to 1933, because Wittgenstein's lectures from this term recorded in the Blue Book, together with the *Brown Book* of 1934, already contain his new methodology and nearly all of the central ideas of his later philosophy as they were to appear in the *Philosophical Investigations*." (West Lafayette: Purdue University Press 1998), 9.

25. G. H. von Wright & B. McGuiness, introduction to *Ludwig Wittgenstein: Cambridge Letters. Correspondence with Russell, Keynes, Moore, Ramsey and Sraffa* (Oxford: Blackwell, 1995), 4. However, G. E. Moore disagrees. In an introductory passage from "Wittgenstein's Lectures in 1930-1933," he writes, "The statement in the Obituary notice in *The Times* for May 2, 1951, that he arrived in Cambridge in 1929 "for a short visit" is very far from the truth." Robert R. Ammerman ed., *Classics of Analytic Philosophy* (New York: McGraw-Hill, 1965), 234 n. 1.

Tractatus Logico-Philosophicus. According to most accounts, a sense that the original project of the *Tractatus* remained incomplete brought him back to academia, not a new philosophical outlook. Nevertheless, as he rethought the *Tractatus*, he realized that nothing less than a radical transformation would do. Consequently, Wittgenstein's philosophical thought evolved rapidly from 1929 to 1933.[26]

Except for some conversations and correspondence with Ramsey[27] and a few members of the Vienna Circle,[28] Wittgenstein isolated himself from philosophy at the completion of the *Tractatus*. Nevertheless, with his return to the academic world, Wittgenstein began to write about philosophy again. The tremendous output of the following years in Cambridge produced two bulky typescripts later published under the title *Philosophical Remarks (Philosophische Bemerkungen*, notebooks composed between

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26. (Shanker 1986, 4) Nevertheless, Wittgenstein's thought evolved in a continuous and gradual way. It suffered no sudden radical shifts. Remarking on the apparently radical differences between Wittgenstein's early and later work, Rush Rhees wrote, "He returned again and again to the question that had occupied him from the beginning. Like anyone else who does this, he came to see difficulties in many of the ideas he had once accepted. In some respects he came to see the problems differently. And as he did so he saw that others methods were needed for the study of them. In all this, I must repeat, he was going more deeply into the problems he had studied at the time when Russell [who had suggested that he threw away his great talent for philosophy in the last twenty years of his life] admired him. If there was anything 'singular' about the changes he made, it was the penetration he showed – the way in which he would recognize difficulties which no ne else would have noticed – and in the persistence with which he discussed the same things". Rhees, Rush, "Ludwig Wittgenstein: a Symposium. Assessments of the Man and the Philosopher" (*The Listener*, IV January 28 and February 4, 1960), 208. Republished in John V. Canfield, *The Philosophy of Wittgenstein: Vol. 4: The Later Philosophy – Views and Review* (New York:Garland Publishing, Inc., 1986), 106.

27. Ramsey visited Wittgenstein for the first time in 1923, with the purpose of correcting his English translation of the *Tractatus*. They soon became friends. Their frequent conversations and correspondence inspired much of Wittgenstein's post-*Tractatus* ideas on mathematics. They remained friends until Ramsey's premature death at age 26, in January 1930. (Sluga 1996, 18)

28. In his "Intellectual Autobiography," Carnap writes that, "In 1927 Schlick became personally acquainted with Wittgenstein. Schlick conveyed to him the interest of our Circle [the Vienna Circle] in his book [the *Tractatus Logico-Philosophicus*] and his philosophy and also our urgent wish that he meet with us and explain certain points in his book which had puzzled us. But Wittgenstein was not willing to do this. Schlick had several talks with him; and Wittgenstein finally agreed to meet with Waismann and me. Thus the three of us met several times with Wittgenstein during the summer of 1927. . . I regretted it when he broke off the contact. From the beginning of 1929 on, Wittgenstein wished to meet only with Schlick and Waismann, no longer with me or Feigl, who had also become acquainted with him in the meantime, let alone with the Circle." (Schilpp 1963, 25-27)

February 1929 and July 1930, first published in 1964*)* and *Philosophical Grammar (Philosophische Grammatik*, written between 1932 and 1934, first published in 1974). Despite being virtually finished works, Wittgenstein did not publish either of them. The need to obtain a research fellowship at Trinity College at the end of 1930 forced him to write the typescript now published as *Philosophical Remarks*. Bertrand Russell reported to the Council of Trinity College, which was considering the award,

> The theories contained in this new work of Wittgenstein are novel, original, and indubitably important. Whether they are true, I don't know. As a logician, who likes simplicity, I should wish to think that they are not, but from what I have read of them I am quite sure that he ought to have an opportunity to work them out, since when completed they may easily prove to constitute a whole new philosophy.[29]

Other important sources for the study of Wittgenstein's thought during this period are Moore's and Lee's[30] lecture notes from 1930 to 1933, and the *Big Typescript* from 1933. According to Hacker's account of Wittgenstein's life at Cambridge,[31] the *Big Typescript* is the closest Wittgenstein came to completing a draft during this period. It is 768 pages long, including an annotated table of contents. Nevertheless, as soon as he 'finished' it, he started making additions, deletions and alterations. These corrections continued sporadically until 1937, the date of the first version of the future *PhilosophicalInvestigations*. This typescript is the last document of Wittgenstein's thought from the middle period. 200 of the remarks in the *Investigations* already occur in it or its revisions.[32]

Even though Wittgenstein did not himself publish any of his ideas during this period, they are not worthless. These writings remained unpublished during Wittgenstein's lifetime, because "he was never quite content with how he had stated his views or ordered

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29. (von Wright 1960, 26 n. 13)

30. Desmond Lee's notes were published as *Wittgenstein's Lectures, Cambridge 1930-1932.*

31. P. M. S. Hacker, *Wittgenstein's Place in Twentieth-Century Analytic Philosophy* (Cambridge: Blackwell, 1996), 79.

32. (Shanker 1986, 86)

the remarks in which they were expressed."[33] On the topic of Wittgenstein's excessively high standards, Norman Malcolm remembers,

> With respect to philosophical work his standards were inexorable. Of a young friend who was preparing a paper to read to the Moral Science Club at Cambridge he remarked that he ought to write it for a hundred years from now and not just for next week. This he said of a paper that was intended merely for a discussion group, not for publication.[34]

Unfortunately, Wittgenstein's philosophy from this period remains relatively unstudied. Testimony suggests that Wittgenstein himself dismissed any other stage in the development of his philosophy besides those of the *Tractatus* and the *Investigations*.[35] As a result, most scholarly work on Wittgenstein's thought either ignores or dismisses the transitional period, focusing instead on the work of the so-called 'early' and 'late' periods.[36] Even the few interpreters who mention it reduce its importance to a mere 'transitional' phase between these two more fully developed stages of Wittgenstein's philosophy. For example, A. C. Grayling writes that "Wittgenstein's position is essentially the same in these as in the chief works" and "The writings of the transitional period are genuinely transitional, containing elements both of the early and the later views."[37] Nevertheless, the importance of studying Wittgenstein's thought during this period goes beyond the mere documentation of the transition from the *Tractatus* to the *Investigations*. Wittgenstein's philosophy in the early thirties is as developed and complete an outlook as that presented in those works.

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33. A. C. Grayling, *Wittgenstein* (Oxford: Oxford University Press, 1988), 64.

34. (Canfield 1986, 105)

35. On *The Conitnuity of Wittgenstein's Thought*, John Koethe writes: "In the preface to the *Investigations*, Wittgenstein remarks of the *Tractatus* that at one time he had thought that he "should publish those old thoughts and the new ones together: that the latter could be seen in the right light only by contrast with and against the background of my old way of thinking (PI x)." (Ithaca: Cornell, 1996), 4. And Norman Malcolm recounts Wittgenstein confiding that he "thought that in the *Tractatus* he had provided a perfect account of a view that is the *only* alternative to the viewpoint of his later work." (Malcolm 1984, 69)

36. Dismissing, also, Wittgenstein's work after the *Philosophical Investigations*.

37. (Grayling 1988, 63)

II. Language, Grammar and Mathematics

Mathematics is no stranger to contemporary linguistics. On the contrary, it has become a very important tool in the scientific study of language. In addition, mathematics is the subject of a considerable strand of linguistic research. For the most part, the linguistic analysis of mathematics is of two different kinds: one which considers mathematics an artificial language,[38] and the other which considers mathematics as part of natural language.

Despite the broad differences between mathematics and natural language, many linguists have found enough *language-likeattributes*[39] in mathematics to justify a linguistic analysis. This sort of linguistic analysis assumes that the syntax and semantics of mathematical statements resembles those of natural language, declarative statements. This justifies applying linguistic tools and theories to the study of mathematics. These linguists see mathematics as an artificial language, independent of natural language. Sometimes, the exaggeration of the separation between mathematical and natural language renders translation impossible.

> Mathematicians know this [that mathematics and natural language are too different to translate from one to the other]. Yet they feel ever the compulsion to interpret their mathematics in terms of the every-day language. So proceeding, their harvest is super-paradox.[40]

Less frequently, linguists consider mathematics either part of or derived from natural language. They provide Chomsky-style transformational grammars for the system of numerals[41] or number names.[42] These linguists consider numerical systems closed regions

38. A. F. Bentley, *Linguistic Analysis of Mathematics, (Bloomington: Principia Press,1932)*

39. C. F. Hockett, *Language, Mathematics and Linguistics* (La Hague: Mouton, 1967), 6.

40. (Bentley 1932, viii)

41. James R. Hurford, *The Linguistic Theory of Numerals* (Cambridge: Cambridge university Press, 1975)

42. Raoul Chapkis & Hugo Brand-Corsitus eds., *Grammars for Number Names* (Dordrecht: Reidel, 1968)

within natural languages.[43] In other words, they contemplate numerals only in relation to other numerical expressions. They ignore the occurrence of numerical terms in natural language. For example, the word 'three' does not interest them in expressions like 'there are three computers in this lab', but only as part of the system of expressions – 'one', 'two', 'three', etc. – for counting. "The ways, that is, in which people in various parts of the world count with words" interests them.[44] Accordingly, most of these analyses belong to comparative linguistics. Moreover, at the foundation of their studies lies the idea that "the notion of numeration and the concepts of particular numbers are universals, and that the linguistic theory must contain the means for describing how each particular language associates arbitrary phonological sequences (words) with these universal concepts."[45]

For this sort of linguistic analysis, mathematics is both a tool for linguistic analysis and part of the very language which it analyzes. Wittgenstein, in turn, puts mathematics inside the grammar of language. For him, mathematics is not only a portion of language, but also a component of linguistic grammar. Mathematics is instrument, subject and result of linguistic analysis.

According to Wittgenstein, traditional linguists incorrectly look for a single grammar of language, ignoring the essential multiplicity of linguistic usage. In doing so, grammarians have concentrated on certain uses, while completely ignoring others. Instead of a single grammar, Wittgenstein encourages looking for many grammars corresponding to the many uses of language. One of these grammars is mathematics. For Wittgenstein, the many systems of rules that constitute *mathematics* are nothing but the grammars of diverse linguistic practices. Geometry, for example, is the grammar for describing objects in visual

43. Barron Brainerd, "On the Syntax of Certain Classes of Numerical Expressions" (Chapkis & Brandt 1968, 9)

44. (Hurford 1975, 2)

45. Ibid.

space. Elementary arithmetic, on the other hand, is the grammar of calculating with numbers. Wittgenstein hopes to dissolve most problems on the foundations of mathematics by exposing their grammatical natures. In particular, he hopes to undermine the puzzles behind the applicability and apparent generality, necessity and normative nature of mathematical propositions.[46]

46. Within the large bibliography on the intersection between mathematics and linguistics, the idea of mathematical propositions as grammatical is not completely foreign. Consider the following passage from the Preface to Charles F. Hockett's *Language, Mathematics and Linguistics*. "Learning mathematics is like learning any subject, in that one must acquire a new vocabulary. It is like learning a foreign language rather than, say, history, in that one must also acquire alien grammatical habits. And it is like no other subject in that one must also learn how to invent *new* grammatical devices as they are needed."