The Problem of Logical Constants and the Semantic Tradition: From Invariantist Views to a Pragmatic Account*

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Abstract: I offer a renewed critique of invariantist views of logicality, objecting especially to Gila Sher's arguments for descriptive adequacy and for invariantism's ability to explain the formality, necessity, apriority and normative force of logic. I also question Sher's idea that the notion of invariance under bijections is theoretically fruitful as a reconstruction of logicality. I argue that the semantic conception of logic can do perfectly well without a model-theoretic notion of logicality, and that the descriptive and explanatory theoretical roles sometimes ascribed to invariantism can be played by a non-model-theoretic account of logicality, specifically by one in which some pragmatic properties of expressions play an important role. I also develop the pragmatic account beyond the mere hints offered in its defense in my earlier work.

1. Introduction

While the problem of logical constants cuts across lines separating the semantic or model-theoretic tradition in logic from other traditions, in the model-theoretic tradition the problem has been felt with special intensity. This is due mainly to the fact that the notion of a logical constant appears explicitly in the general formulation of Tarski's (1936) method for the construction of definitions of logical consequence, even if a characterization of the notion is not needed for the construction of particular definitions, which can simply rely on particular lists of logical constants. Although there are several characterizations of logical constancy within the model-theoretic tradition, all share the idea that the logicality of an expression is to be understood as consisting in some kind of invariant behavior across some kind of mathematical transformations between some kind of domains. Among the corresponding notions we find the notions of invariance under permutations (Tarski and Givant (1987), ch. 3, Keenan and Stavi (1986)—prefigured in Tarski (1966), though not strictly speaking as a notion of logical constancy[1]), of invariance under bijections (van Benthem (1986), ch. 2), of invariance under surjective functions of a certain kind

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[1] Even in Tarski and Givant (1987), the notion of invariance under permutations is used basically stipulatively, and not really offered as a reconstruction of the pre-theoretical concept of logical constancy.

(Feferman (1999), (2010)), of invariance under arbitrary surjective functions (Casanovas (2007)), and of invariance under potential isomorphisms in a certain sense (Bonnay (2008)). All these are notions definable in terms of the mathematics of classical model theory, as supplemented with appropriate definitions of the concepts of denotation of an expression in a domain assumed by the different invariance notions. There are also other, less purely mathematical invariance notions that have been proposed as reconstructions of the notion of logicality, such as the notions of rigid invariance under various modalities (metaphysical, epistemic, etc.) in McCarthy ((1981), (1987)), the notion of invariance under bijections of domains containing both actual and fictional individuals in Sher ((1991), (2003), (2008)), and the notion of analytical invariance under bijections in McGee (1996).

Among all these notions of invariance, Sher's notion of invariance under bijections (often considered in abstraction from its differences with respect to the more purely mathematical notions of invariance under permutations and of invariance under bijections) has some right to be considered the most popular invariance notion of logicality, and has even been called the "received" view in the semantic tradition (Bonnay (2014), 54). I would hesitate to call Sher's view the received one, even in the semantic tradition,[2] but the notion of invariance under bijections (at least in abstraction from the differences between the related more purely mathematical notions and Sher's notion) is probably the most well-known reconstruction of logicality in the semantic tradition. And it is in any case the invariance under bijections theory of logicality that has been more ambitiously and vigorously defended from a philosophical point of view within the tradition, especially by Sher, but also by other recent defenders (see, e.g., Sagi (2015), Griffiths and Paseau (2016) and Woods (2016)). In a critical appraisal of invariantist ideas about logicality, it is thus natural to focus on the invariance under bijections idea as presented by Sher and on her defense of it (whether or not we consider it in abstraction from the mentioned differences between it and other notions), and this is what I will do in the present paper. Nevertheless, it will be easily seen that many of the critical points leveled at Sher's notion of invariance under bijections as a view of logicality in what follows can also be reformulated as points against other invariance proposals.

[2] I would guess that the vast majority of logicians and philosophers in the tradition simply have no definite view on the matter of logicality, as opposed to the situation with regard to, say, what is the appropriate model-theoretic notion of logical consequence for classical quantificational logic or what is the right notion of computable function.

Sher often presents her proposal as having a strongly prescriptive component and appeals to the fruitfulness of the notion of invariance under bijections in logic and linguistics as one of the main justificatory elements for the proposal. To some extent, proposing a definition as prescriptive shields it from philosophical criticism. However, as we will see, Sher is also much concerned with defending the descriptive adequacy of her proposal and with offering arguments that the proposal is explanatory of a number of philosophically important properties of logical truth. In this paper, I will begin by criticizing Sher's arguments for descriptive adequacy (section 2) and for explanatory richness (section 3), and will also question her idea that the notion of invariance under bijections is theoretically fruitful *as a reconstruction of logicality* (also in section 3). This critique leaves one wondering what is to be made of the problem of logical constants from the point of view of a sympathizer of the semantic conception of logic. The final section 4 will suggest that the semantic conception can do perfectly well without a model-theoretic notion of logicality, and that the descriptive and explanatory theoretical roles ascribed to invariance under bijections by Sher can be played by a non-model-theoretic account of logicality, and specifically by one in which some pragmatic properties of expressions play an important role. I will also develop the pragmatic account somewhat beyond the mere hints offered in its defense in my earlier work.

2. Invariance under bijections and descriptive adequacy

In the literature there are two main sorts of considerations against the descriptive adequacy of the invariance under bijections idea.[3] Both accuse invariance under bijections of allowing

[3] I should perhaps make explicit the notion of invariance under bijections at stake. Leaving aside for the moment complications having to do with the nature of the domains involved, let's suppose we have a class of domains we are focusing on. If U and V are domains of the same cardinality and B is a bijection between them, let B^+ be the bijection induced by B between the objects of the hierarchy of types generated from U and the objects of the hierarchy of types generated from V. (B^+ can be defined recursively over the class of objects in the hierarchy generated from U thus: $B^+(\mathrm{o})=B(\mathrm{o})$ if $\mathrm{o} \in U$; otherwise $B^+(\mathrm{o})=\{B^+(\mathrm{p}): \mathrm{p} \in \mathrm{o}\}$.) Let's also use the notation $den(C, U)$ for the denotation of an expression C with respect to a domain U. Then a constant C is invariant under bijections (with respect to the given class of domains) just in case for all universes U, V of the same cardinality (in that class) and every bijection B from U onto V, $B^+(den(C, U))=den(C, V)$. (The denotation of a first-order quantifier in a domain can be seen in the Fregean fashion, as a subset of the power set of the domain. The denotation of a connective in a domain can be seen as a function from tuples of sets of assignments to the variables to sets of such assignments. (Sher, however, deals with the connectives as a special case, stipulating that a connective is logical just in case it is truth-functional.))

too many constants to be logical. The first sort of consideration is that a lot of constants that appear to have a rich set-theoretical content are invariant under bijections, such as cardinality quantifiers like 'There are infinitely many things such that' and 'There are uncountably many things such that'. These constants strike many logicians and philosophers as non-logical in some intuitive sense, despite being invariant under bijections. This fact has actually motivated some of the other proposals mentioned above that attempt to characterize logicality in terms of invariance under larger classes of transformations, which makes it more difficult for a constant to be declared logical. (I'm thinking especially of Feferman's[4] and Bonnay's proposals, the former of which excludes both 'There are infinitely many things such that' and 'There are uncountably many things such that', and the latter of which excludes 'There are uncountably many things such that'.) Sher's reaction to these worries has been to adopt the thesis that mathematics and logic share a common set-theoretical core which provides a foundation for mathematics and logic, and which includes all the invariant notions (see, e.g., Sher (2008), 318ff.). This reaction, which amounts to ascribing both a logical and a mathematical nature to set-theoretical notions, seems to me reasonable, insofar as the modern tradition in logic has often treated large parts of classical mathematics, and sometimes even the high reaches of set theory, as a part of logic, and there is no established consensus that they should not be so treated. Quantifiers such as 'There are infinitely many things such that' and 'There are uncountably many things such that' are simply not fully clear cases of non-logical constants, or cases of non-logical constants for every pre-theoretical understanding of logical constancy. The semantic tradition has typically considered acceptable those theoretical reconstructions of a concept that include all pre-theoretical clear cases of the

Sher normally speaks of invariance under isomorphisms rather than of invariance under bijections, though she makes it clear that she assigns denotations to constants merely as a function of the relevant domain, the further structure playing no role in this; and she sometimes notes that the terminology of 'invariance under bijections' might be used just as well (see e.g. Sher (2008), 302ff.). As Sher notes, her use of 'invariance under isomorphisms' derives from Lindström's requirements on generalized or abstract logics, one of which is precisely that a sentence of an abstract logic have the same truth value in all isomorphic structures. (In Lindström's framework, however, predicate and function constants (aside from identity) are stipulated to be non-logical in general; see below.) I will stick to 'invariance under bijections' in what follows.

[4] It ought to be mentioned that in what was perhaps his last publication on these issues, Feferman abandoned his earlier invariantist proposal and put forward a mixed semantic-inferentialist proposal that would seem to go strictly beyond the limits of the semantic tradition; see Feferman (2015). This awaits detailed examination elsewhere.

concept and exclude all pre-theoretical clear non-cases, while leaving much room to discretion concerning pre-theoretical borderline cases of the reconstructed concept.[5] Many set-theoretical constants, however rich or sophisticated in their content, may well simply be pre-theoretical borderline cases of logicality.[6]

The second main sort of consideration against the descriptive adequacy of invariance under bijections, however, is that a lot of non-set-theoretical constants which in some intuitive sense seem clearly non-logical are invariant under bijections. To take three examples from my earlier work (Gómez-Torrente (2002), (2003)), 'unicorn', 'heptahedron' (meaning "regular polyhedron of seven faces") and 'male widow' have the empty extension in all domains, or at any rate in all actual domains, in all metaphysically possible domains,[7] and in other sorts of domains as well, and are thus invariant under bijections of these domains. However, they would seem to be pre-theoretical clear cases of non-logical expressions.[8] (Needless to say, the three examples just mentioned are not isolated cases, emptiness of extension is not a necessary feature of this kind of counterexamples, and counterexamples of this kind exist in all syntactic categories. To take just two more examples illustrating this, 'non-unicorn' will be invariant under bijections of non-empty

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[5] Nevertheless, the fact that Sher's proposal definitely declares logical many constants which are borderline cases points to a limitation of the proposal that the pragmatic account below doesn't have, as I will emphasize in section 4.

[6] A related but different complaint, which doesn't seem to have worried too many people in the semantic tradition, is that there are purely extensional expressions that seem (somewhat) logical in some intuitive sense but are not invariant under bijections; the predicates 'is a part of' and 'is true' are examples. (But see Cook (2012) and (forthcoming) on 'is true' and 'is valid' and Woods (2014) on "logical indefinites".) However, a defender of invariance under bijections might allege that these are not clear or definite cases of logical constants. We will come back to these examples in section 4 below. (For discussion of yet a different complaint, namely that some non-extensional but intuitively logical expressions, in particular modal operators, come out non-invariant under bijections (under suitable assumptions) see e.g. Novaes (2016).)

[7] 'Unicorn' will be declared to have an empty extension in all actual and metaphysically possible domains if the widely accepted Kripkean view of its meaning and the way its reference is fixed is correct. This will be so whether we assume a "naïve" realism about possible worlds on which these have some kind of real existence aside from their representations (and thus that the models built out of existents in a possible world have the same sort of real existence), or whether we take talk of possible domains (and possible models) as a mere *façon de parler*.

[8] In the case of 'heptahedron', it may be worth noting explicitly that geometry has been one of the parts of classical mathematics that have not been typically treated as parts of logic by the modern logical tradition.

domains; and a quantifier '$\exists^{\text{male-widow}}$', defined to be synonymous with 'nothing is a male widow and there are things such that..., or something is a male widow and everything is such that not...', will have the same extension as the existential quantifier in all domains, and will thus be invariant under bijections.) Assuming the examples in question are indeed pre-theoretically clear cases of non-logical expressions (and as far as I know, no one has denied this in the literature on the topic), invariance under bijections has a serious descriptive problem given the semantic tradition's typical requirement for theoretical reconstructions of a concept, that they should exclude all pre-theoretical clear non-cases of the concept.

Before moving to Sher's reaction to this second main sort of consideration against descriptive adequacy, it's good to note that the examples on which it is based are substantively different from other non-set-theoretical examples considered in discussions of the issue of descriptive adequacy. Sagi (2015) has considered some examples that she finds in Hanson (1997) and McCarthy (1981) and has noted that they cannot be used against Sher's proposal. Hanson's example as he presents it is this:

> let n be the least number of whole seconds (that is, the least number of seconds, disregarding fractions of a second) in which, up through the end of the twenty-first century, a human being runs a mile. Now consider a quantifier that behaves exactly like the universal quantifier (over individuals) in models with domains of cardinality ≥n, but like the existential quantifier in models with domains of cardinality <n. Call this quantifier 'Q*'. 'Q*' is a logical term on Sher's account because it satisfies her semantic isomorphism conditions, although it seems bizarre to treat it as one. To see just how bizarre this is, consider the following argument:
>
> (7) (Q*x) (Dog(x) → Black(x))
> (Q*x) Dog(x)
> ∴ (Q*x) Black (x)
>
> As long as n≥3, argument (7) has countermodels. So we know that (7) is invalid, since we know that no one will run a mile in less than three seconds before the end of the next century (or ever, for that matter). Yet we don't and can't know this a priori. (Hanson (1997), 391-2)

The idea would seem to be that an argument that is logically invalid in the model-theoretic sense if we assume Sher's criterion for logical constancy is nevertheless not knowable *a priori* to be invalid, while logically invalid arguments should be *a priori* knowable to be invalid, giving us reason to see Sher's criterion as the probable culprit.[9]

McCarthy's example as he presents it is this:

[9] I think this is not a good criticism of Sher, independently of anything discussed below. While I think it may be reasonable to demand of a *valid* argument that its validity be in some sense *a priori* knowable, I don't think it's reasonable to demand of an arbitrary *invalid* argument that its invalidity be *a priori* knowable, or just knowable, for that matter. For Hanson's purposes, it would be better to define n as (e.g.) the number of natural satellites of the Earth, and then say that we know, but cannot know *a priori*, that (7) is valid.

> Suppose that '*K*' represents a certain *a posteriori contingently true* statement of the metalanguage, and consider a one-place sentential connective 'N' characterized by
>
> (7) $\forall s$ (s satisfies $\ulcorner N\phi \urcorner \leftrightarrow [(\sim s$ satisfies $\phi \wedge K) \vee (s$ satisfies $\phi \wedge \sim K)])$
>
> (7) provides that the extension of a formula $\ulcorner N\phi \urcorner$ over [a domain] $\mathscr{L}$ is that of ϕ if $\sim K$, and its complement with respect to $\mathscr{L}$ if K. (McCarthy (1981), 514)

McCarthy then says that "in any counterfactual situation in which a sentence ϕ is true but in which '*K*' is false, the sentence $\ulcorner N\phi \leftrightarrow \sim\phi \urcorner$, though a logical truth, is false" (McCarthy (1981), 515). $\ulcorner N\phi \leftrightarrow \sim\phi \urcorner$ is a model-theoretic logical truth if 'N' is a logical constant (which it is on the invariance under bijections criterion), as '*K*' is actually true, and yet $\ulcorner N\phi \leftrightarrow \sim\phi \urcorner$ is not necessary, because (McCarthy implies) it is false in worlds where '*K*' is false; this is once more supposed to give us reason to doubt the invariance criterion, again under the assumption that the model-theoretic account of logical consequence and logical truth is not under dispute.

Sagi reads Hanson and McCarthy in these texts as giving merely instructions for determining the extensions of 'Q*' and 'N' in actual domains, not definitions of them in terms of synonymous expressions, or even instructions giving their extensions in domains from arbitrary possible worlds or epistemic situations, and attributes their idea that those instructions lead to epistemic or modal problems to a fallacy. She says, for example, that McCarthy's argument

> relies on an invalid inference from a modal statement about a linguistic entity, namely "The sentence '$N\phi \leftrightarrow \sim\phi$' may have been false" to the modal status of the content of that linguistic entity, "It may have been that $N\phi \leftrightarrow \sim\phi$ was not the case." Now, if '*K*' were not true, then '$N\phi \leftrightarrow \sim\phi$' would not have been true: merely because we would have attached to '*N*' a different meaning. (Sagi (2015), 162-3)

If all this is so, Sagi notes, we have not been given good arguments that 'Q*' and 'N' are not logical constants: '*K*' is an expression of McCarthy's metalanguage which "is not re-interpreted or re-evaluated itself" (Sagi (2015), 164), and certainly, if '*K*' is playing no role in assigning counterfactual truth conditions to '$N\phi \leftrightarrow \sim\phi$', then we have no way of evaluating the sentence in other possible worlds and no way of telling whether it is necessary. This leaves room for deeming it necessary, and according to Sagi it is in fact necessary, because she adopts the view that actual models faithfully represent possible worlds, and so model-theoretic logical truth is coextensive with necessity for sentences whose necessity is due to the content of the logical constants.

I think it's not entirely transparent textually that Hanson and McCarthy see their instructions for interpreting 'Q*' and 'N' as mere instructions for determining the extensions of 'Q*' and 'N' in actual domains. (In fact, I think it's reasonably clear from the context of McCarthy's exposition that he is giving for 'N' a satisfaction condition relative to arbitrary possible worlds in an intensional metalanguage, even if he is not making this explicit in the notation.[10] If this is so, then McCarthy is right to say that '$N\phi \leftrightarrow \sim\phi$' is a model-theoretic logical truth even though it is contingent, as it inherits its contingency from the contingency of the metalinguistic 'K', and no fallacy of interpreting 'N' with a different meaning in different counterfactual circumstances is involved.[11]) But there is no doubt that Sagi's substantive non-exegetical point is correct; that is: *if* we are only given a material satisfaction condition for an expression which implies that it is invariant under bijections, then there will still be little we can conclude (non-fallaciously) about its status as logical or non-logical. There will be expressions (full-fledged expressions, with all aspects of their content settled) coextensional with the condition and intuitively non-logical and expressions coextensional with the condition and intuitively logical (as our following discussion will make clear).

By contrast with 'Q*' and 'N' with their associated instructions interpreted as material satisfaction conditions, however, 'unicorn', 'heptahedron' and 'male widow' are full-fledged expressions, with all aspects of their content settled (or so we can presume). Their intuitive non-logicality is then carried by them on their sleeves, as it were. But in some cases their non-logicality may also be manifested via modal, epistemic, or other tests. Thus (as noted in Gómez-Torrente (2002), 19) '$\forall x \sim$unicorn(x)' (regardless of what domain we interpret its quantifier as ranging over) is not intuitively a logical truth, presumably in part because it is not *a priori* (even if it is necessary), even though it is a model-theoretic logical truth if 'unicorn' is taken as a logical constant; '$\forall x \sim$heptahedron(x)' is not intuitively a logical truth (even if it is necessary and *a priori*), presumably in part because it is not analytic, even though it is a model-theoretic logical truth if 'heptahedron' is taken as a logical constant; and '$\forall x \sim$male widow(x)' is not intuitively a logical truth (even if it is necessary, *a priori* and analytic), presumably in part because somehow it doesn't have the right kind(s) of necessity, apriority or analyticity.

[10] That's why he says that "since (7) is presumed to state the satisfaction *condition* of sentences of the form '$N\xi$', the predicate 's satisfies $\ulcorner N\phi \urcorner$' *rigidly* coincides with the predicate '($\sim s$ satisfies $\phi \wedge K$) $\vee$ (s satisfies $\phi \wedge \sim K$)'" (McCarthy (1981), 515). I don't see how to understand this unless McCarthy is thinking of his instruction as intensional.

[11] See Woods (2016), for another charitable way of understanding the Hanson and McCarthy examples.

Sagi mentions the problem posed by these examples from my earlier work,[12] but defends the invariance proposal from them, attributing the problem to

> a limitation of the framework of extensional logic. Whatever can be represented by arrays of extensions is the most of the meaning of a term that can be captured in this framework. …any difference between two metalinguistic rules giving the same array of extensions is irrelevant to the modal status of sentences in the object-language. (Sagi (2015), 163)

The suggestion is perhaps implicit that the framework of extensional logic is uncontroversial and elucidatory of uncontroversial aspects of the content of an expression, while any theoretical apparatus that could sieve out 'male widow', say, from the analytically coextensional but intuitively logical 'is different from itself' would be controversial. Relatedly, Bonnay speaks of this kind of examples as pointing "at problems in our conception of interpretation and meaning" ((2014), 60), suggesting that extension is the core of content, and other aspects of content such as those presumably responsible for the intuition that 'male widow' is non-logical will be fraught with controversy.[13] To someone taking the examples as dismissible on account of the fact that they involve non-extensional aspects of the content of some expressions, there is little I can say, except that if we take

[12] She also attributes an example similar to 'male widow' to McGee (1996), 578, but the McGee example she refers to is actually not similar in the relevant way. The example is a unary connective 'H' such that 'Hϕ' is defined to be synonymous with '$\sim\phi \wedge$ water is H_2O'; this is dissimilar in that 'H' is not invariant under bijections in virtue of its meaning, while 'male widow' is. (On the usual view, the truth of 'Water is H_2O' (if it is true) is not due to its meaning, but to some aspect of the metaphysical essences of water and H_2O.)

[13] On the other hand, Bonnay speaks of these examples as showing that invariance (in some version) is a "starting point" ((2014), 60) for a theory of logical constancy, even if it doesn't provide the whole story. If this is all that Bonnay means, I need not disagree with him.

Bonnay also appears to float the idea that a logical expression is an expression that is invariant under some appropriate class of transformations (say bijections, for definiteness) and is also *directly referential* (*ibid.*). I would object to this idea as well. The idea certainly excludes as logical expressions which are invariant under bijections but are synonymous with more complex expressions. However, suppose we stipulate 'heptahedron*' to be a directly referential expression whose reference is fixed to be that of 'heptahedron' (being directly referential, it will have the same reference in all possible worlds). 'Heptahedron*' is intuitively non-logical, insofar as the only information we have about the fixing of its reference (which is *a priori* information) employs non-logical resources, even though it is necessarily and *a priori* invariant under bijections. As another shot, consider the hypothesis that a logical expression is one that is both invariant under bijections in virtue of its meaning *and* directly referential. This gets rid of the problem with 'heptahedron*', but would in any case exclude expressions defined to be synonymous with complex expressions that we want to consider as logical, such as 'non-self-identical'. (Bonnay's proposal that a logical expression is one that is invariant and directly referential has this problem as well.)

this route we are simply denying the existence or importance of intuitive distinctions, but we have no convincing reason to consider these distinctions as unreal or as not grounded on real and important underlying linguistic or extra-linguistic facts. Flatly denying an intuitive aspect of reality is not typically the right philosophical way to go, or it doesn't strike me and many others as right, at any rate.

Now while Sher herself occasionally speaks as if the examples in question would be somehow unimportant even if they were to some extent correct (see, e.g., Sher (2013), 178), she does want to claim that they do not provide a correct objection to her theory in any sense. According to her, 'unicorn', 'heptahedron' and 'male widow' are not declared logical constants by her proposal: "The correctness of Gómez-Torrente's criticism thus depends on whether 'unicorn', 'heptahedron' and 'male widow' (…) are logical constants according to my proposal. I will show that they are not" (Sher (2003), 193). How could this be so, if those predicates clearly have the empty extension in all actual domains and in all metaphysically possible domains?

The first reason why Sher says that 'unicorn', 'heptahedron' and 'male widow' are not logical constants according to her theory is that they are not defined over all domains, while her characterization of logical constants explicitly requires that a logical constant be defined over all domains. She illustrates the point with 'unicorn':

> 'Unicorn' (as it is used in English) is a zoological predicate, and as such it is inapplicable to non-zoological, or at least to non-biological, objects—numbers, thoughts, planets, etc. It follows that the meaning of 'unicorn' would not be accurately captured by a function defined over models with universes consisting of (non-empty) sets of such objects. Hence construing 'unicorn' as a logical constant would violate condition (D) [the condition in Sher's characterization that explicitly requires that a logical constant be defined over all domains]. (Sher (2003), 193)

This reaction strikes me as strange. What makes it the case that 'unicorn' ("as it is used in English") is defined or undefined over a domain? Presumably the intuitions of speakers of English. Is it intuitively defined whether the thought I just had about Chapel Hill being north of Mexico City is a unicorn? I would say that it is clearly defined, and that it is defined that that thought is *not* a unicorn.

But it is conceivable that this counter-reaction could be considered a bit shaky, insofar as it relies on the much maligned speaker intuitions—perhaps Sher and a substantial number of others believe that it is unclear whether my thought that Chapel Hill is north of Mexico City is a unicorn ("as 'unicorn' is used in English"). Another counter-reaction that should be more worrying for Sher, however, is that if the reason why 'unicorn', 'heptahedron' and 'male widow' are not logical constants according to her theory is that they are not defined over all domains, then there is something fishy about the theory's way of discriminating the logical from the non-logical constants. If 'unicorn', 'heptahedron' and 'male widow' are excluded from logical constancy because they are not defined over all domains, presumably the same is the case for the vast majority of expressions, which would thus be excluded not on account of their not being invariant, but on account of their not being universally defined. What would be doing the job of discriminating the logical from the non-logical constants, then, would be, for the vast majority of expressions, the question about their being universally defined or not. And then the notion of invariance would be playing little or no role in the demarcation of the logical constants after all; intuitions about whether particular expressions are defined over all domains would play the basic role. An indication that this is not really what Sher (at least originally) wanted or intended is the way she argues that some constants that would seem to be not universally defined according to her (later) intuitions are not logical constants: this is not by observing that they are not universally defined, but by noting that they are not invariant under bijections, which presumably *presupposes* that they are defined over all domains. Thus, for example, when Sher argues that the quantifier " 'pebbles in the Red Sea', defined by $f_{\text{pebbles}\ldots}(\mathbf{A})\{B\text{: } B \subseteq A \text{ \& } B$ is a nonempty set of pebbles in the Red Sea}" (Sher (1991), 58) is not a logical constant on her theory, she does so by claiming that it is not invariant under bijections, although presumably the quantifier in question is not defined over "non-mineralogical" domains according to her (later) intuitions.

In any case, and even if we leave aside this (kind of *ad hominem tu quoque*) counterobjection, the objection that 'unicorn', 'heptahedron' and 'male widow' are not defined over all domains simply postpones the difficulty for Sher. As I noted in Gómez-Torrente (2003), we can define in a simple way new terms, 'unicorn#', 'heptahedron#' and 'male widow#' that are defined over all domains and are still intuitively non-logical. Define 'unicorn#' as synonymous with 'either a zoological individual and a unicorn or else a thing non-identical with itself'. 'Unicorn#' has the same extension as 'unicorn' in domains where 'unicorn' is defined (according to Sher's intuitions), and the empty extension in other domains. 'Heptahedron#' and 'male widow#' can be introduced similarly. If 'unicorn' has the empty extension in those domains where it is defined, as seems clearly to be the case, then 'unicorn#' will be invariant under bijections, despite the fact that it is a pre-theoretically clear case of a non-logical constant.

In her (2003) Sher actually considers examples essentially similar to 'unicorn#', 'heptahedron#' and 'male widow#', and again claims that they are not logical constants according to her theory. In the case of 'unicorn#', she says that this is so because some domains include a non-empty subset of unicorns, to wit, domains containing fictional or mythological unicorns;[14] in such domains 'unicorn#' (and 'unicorn') will not have the empty extension. Again this seems to reveal a change of heart, for Sher (1991) gave no indication that she was basing her theory on a class of domains different from the domains usual in mathematical model theory, namely sets built out of actually existing urelements; in sets built out of actually existing urelements there are no (fictional) unicorns (or heptahedra or male widows), to be sure. If her domains do contain unicorns, then her notion of invariance is not really a purely mathematical notion along the lines of Tarski, Van Benthem, Feferman and Bonnay, and is instead a non-purely mathematical notion that involves in turn a potentially precarious notion of domain.

To see just how precarious the needed notion is, we simply have to note that there are hardly any limits to the properties that fictional objects could possess. Fictions can be inconsistent, for example, and so can be fictional objects. Nothing prevents a fiction about an object which is both red and green (and hence non-red), or about a set that contains all and only the sets that do not contain themselves, or about a set that contains both exactly seven and exactly seventeen objects, or indeed about an object that is non-identical to itself. But if such objects form part of the domains that play a role in our model-theoretic reconstruction of semantic notions, then our theory of logical consequence and logical truth will again be vitiated. Regardless of whether we take 'unicorn' or 'unicorn#' as a logical constant or not, and provided merely that, say, the identity symbol is a logical constant, '$\forall x(x=x)$' will not count as a logical truth, as there will be domains of fictional individuals where some object is non-identical to itself.

Sher says that basically the same move as with 'unicorn#' is to be made in the case of 'heptahedron#' and 'male widow#' (see Sher (2003), 195); that is, we are to suppose that there are domains containing heptahedra# and male widows#. But here the idea seems to be somewhat different:

> we overcome the problem [that 'heptahedron' or 'heptahedron#' has always the empty extension and is thus indiscernible from 'non-identical to itself'] by constructing $f^*_{heptahedron}$ as a function that "regards" one (conventionally designated) object with no geometric properties, *b'*, as a heptahedron in all models to which it belongs. In that case, however, [the invariance under bijections condition] is violated. (Sher (2003), 195)

[14] In Sher's words: " 'unicorn' has a non-empty extension in some models (namely, models whose universes contain mythological animals of the kind unicorn)" (Sher (2003), 194).

And she adds that "for similar reasons 'male widow' is not a logical constant".[15] I find this just as much troubling as the appeal to fictional entities in the case of 'unicorn' and 'unicorn#'. If we can "conventionally designate" some objects as members of the extension of a term (with respect to a domain) when that suits our purposes, there is no objective, non-*ad hoc* reason why 'heptahedron#' (or 'heptahedron', or 'male widow', or 'male widow#') is not a logical constant on Sher's theory. There is nothing to prevent someone from using the theory, via the same *ad hoc* maneuver, in order to claim that 'non-identical to itself' is not a logical constant: it will be enough to "conventionally designate" an object that is non-identical to itself in all domains to which it belongs.[16]

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[15] She also says that "'heptahedron' in English is well-defined over one-sided surfaces in accordance with Hilbert & Cohn-Vossen's use (1952: 302-3), so that 'heptahedron' has a non-empty extension in some models" (Sher (2003), 195). But this is evidently irrelevant to the objection at stake here, for in Hilbert and Cohn-Vossen's (1952) use, 'heptahedron' doesn't mean "regular polyhedron of seven faces", as it does in our discussion. They use 'heptahedron' as a name for a certain kind of one-sided irregular polyhedron whose seven faces are four triangles and three squares.

[16] This is somewhat related to van Benthem's criticism of invariance proposals as "blatantly circular" (van Benthem (2002), 428). By this he means that invariance proposals "must make a number of prior decisions that manipulate the final outcome: *Which objects are taken? And then, which transformations?*" (428-9). Note that, even if there are several options as to what objects can appear in the domains mentioned by an invariantist proposal, and as to what kind of transformations between domains is to be used in the proposal, there is no problem *per se* in fixing these things from the start and see to what results they lead us. Surely any theory about any subject matter "must make a number of prior decisions that manipulate the final outcome"; that's what theorists' work is all about. What is problematic are the results of some of these choices, as illustrated in the main text in the case of the choices made by Sher. In p.c., van Benthem has emphasized to me that the worry he has in mind is that the usual formulation of invariance is insufficiently general, as it is "undefined" for mass quantifiers or for modal expressions (compare Novaes's remarks cited in an earlier note). (It is then doubtful to me that "circularity" was the right word for the idea van Benthem has in mind.) He says that this makes invariance unsuitable for a "foundational" account of logicality, although he is not much concerned by this. I myself don't see a problem for invariantism in the insufficient generality of its usual formulation; I suppose that if something appropriately general is provided some day, the usual formulation will be a particular case that will still apply to a restricted range of expressions. (Insufficient generality just means that, as van Benthem points out, more work needs to be done by the invariantist.) But, unlike van Benthem, I am concerned with the "foundational" question whether invariantism has the ability to provide a basic, non-*ad hoc* understanding or justification of our main pre-theoretical ideas about logical constancy. Again assuming that the usual formulation will be a particular case of an eventual completely general formulation, the criticisms in the text provide my own reasons for thinking that invariantism is indeed unsuitable for a "foundational" account of logicality.

However, in the case of 'heptahedron#' and 'male widow#', Sher claims that there is one reason why they are not logical constants according to her theory *even if* we suppose (against her best judgment) that they have the empty extension in all domains:

> In that case, (the reconstructed) 'heptahedron' [i.e., 'heptahedron#'] will not satisfy (B)-(D) [Sher's conditions requiring that a logical constant be identified with its function from domains to extensions (B), defined over domains so as to yield a subset of the domain as extension of a predicate in the domain (C), and defined over all domains (D)]: (B)-(D) require that the definition of ['heptahedron#' capture] its intended meaning in L, i.e., its English meaning. But if we define ['heptahedron#' as above], this requirement is violated. Why? Because in that case the meaning of ['heptahedron#'] in L is the same as the meaning of '∅'[[17]] in L or that of 'non-self-identical' in English. But ['heptahedron#'] is a geometrical concept, and as such its meaning in L (by assumption, its meaning in English) is very different from the meaning of the non-geometrical concept '∅' in L or that of 'non-self-identical' in English. (Sher (2003), 195)

She is thus saying that one reason why 'heptahedron#' is not a logical constant on her theory is that our definition of it does not capture its meaning, while her requirement (B), that a logical constant be identified with its function from domains to extensions, implies this as a requirement. Why is the meaning of 'heptahedron#' not captured by its definition? Because this definition implies that 'heptahedron#''s class-function mapping domains into extensions drawn from those domains (the only thing that can be the "meaning" of 'heptahedron#' in terms of Sher's apparatus) is the same as that of other constants which differ from it in their intuitive meaning, such as 'non-self-identical'.

I think it should be evident that the requirement that the extensional "meaning" class-function of an expression should not be the same as that of other constants (of the same language) imposes a burden on Sher's theory that cripples it nearly completely. First of all, the requirement implies that there will be innumerable languages and expressions within them to which the theory will be inapplicable. If the theory is inapplicable to a language containing both 'heptahedron#' and 'non-self-identical', that's bad enough. But second, and perhaps more importantly from Sher's point of view, the requirement precludes the possibility of languages containing intuitively logical constants which are coextensional and yet differ in their intuitive meaning. Thus, for example, Sher's theory is inapplicable to a language containing both a quantifier synonymous with 'there are things such that' and a quantifier synonymous with 'not everything is such that not', which both have the same associated class-function over domains but intuitively differ in meaning, and yet are clearly logical constants.

As we see, the second main sort of counterexamples to the descriptive adequacy of the invariance under bijections proposal are fairly resilient, and not just because they, or

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[17] In my earlier work I introduced '∅' as an abbreviation of 'non-self-identical'.

complications of them, resist Sher's reactions, but also because they, or complications of them, resist other versions of invariance proposals. Counterexamples of this kind obviously plague also purely set-theoretical proposals such as Feferman's and Bonnay's, insofar as these proposals declare logical constants all the expressions coextensional with the pre-theoretically clear cases of logical constancy that motivate them. But the counterexamples also affect invariantist proposals that use non-set-theoretical resources more openly than Sher's, such as McCarthy's and McGee's. On McCarthy's ((1981), (1987)) preferred idea, a logical constant is an expression which is invariant under bijections of (essentially) "epistemically possible" domains. Perhaps this excludes 'unicorn', if there are "epistemically possible" domains containing unicorns. But presumably 'heptahedron' and 'male widow' have an empty extension over all "epistemically possible" domains—and if McCarthy wants to claim otherwise, then it's unclear why pre-theoretically clear cases of logical constants should not also have non-standard extensions over some "epistemically possible" domains, rendering the proposal *ad hoc*. On McGee's (1996) briefly sketched idea, a logical constant is an expression from whose meaning it follows that it is invariant under bijections (in domains of all kinds, apparently). Presumably this excludes 'unicorn' and 'heptahedron', but not 'male widow'.

The second main sort of counterexamples to the invariance under bijections proposal are important not just because they show that existing invariantist proposals are descriptively inadequate. Perhaps more importantly, they point the way to several skeptical considerations about Sher's ambitious claims that the invariantist idea provides a foundation or explanation for some of the philosophically important properties that logically correct arguments and logically valid sentences appear to possess. In the next section we will give expression to these and other critical considerations of invariantism.

3. Invariance, explanatory power, and theoretical richness

That Sher's claims concerning the explanatory power of her proposal are indeed ambitious is made plain by the following quotation:

> Logic is often characterized by its basicness, generality, topic-neutrality, necessity, formality, strong normative force, certainty, a-priority, and/or analyticity. While, as foundational holists, we reject the purported analyticity of logic and qualify its a-priority, we can explain its other characteristics (including quasi-apriority) based on the Invariance-under-Isomorphism criterion, i.e., explain why the laws of logic and its consequences are as basic, general, topic-neutral, formal, strongly normative, and highly certain as they appear to us to be, and to what degree they are a-priori. (Sher (2008), 316)

(Necessity is omitted in the last enumeration, but this is just an oversight.) We will question, to varying degrees, all the purported explanations offered by Sher, paying closest attention to those of necessity, strong normativity (closely linked to "basicness") and (quasi-)apriority (closely liked to "high certainty").

Sher's claims of explanatory richness are developed most fully in Sher (2008), where she treats logicality as a property of operators, by which she means the meaning class-functions defined over domains which we have seen she identifies with expressions in other parts of her work. (An operator is invariant under bijections when, for any two domains and any bijection between them, the operator's value on the first domain is the image under the bijection of its value on the second.) Sher attributes the formality of logic to the invariance of logical operators. The idea is that invariant operators do not distinguish between bijectable domains, which in a reasonable sense can be said to have "the same form".[18] I have little to object to this use of the Protean words 'form' and 'formal', and if invariance under bijections is indeed a necessary condition of logical operators (or of logical constants),[19] then there is no objection to the claim that logic is formal in the sense that its constants signify (or are identified with) operators which are invariant under bijections. However, this is barely more than a choice of usage, and has little explanatory value in and of itself. The traditional distinction between the form and the matter of an argument or proposition is essentially just the distinction between its logical constants and its non-logical constants. A theory of logicality is thus automatically a theory of formality, and so there is little to be objected to the trivial claim that a certain theory of logicality is a (good) theory of formality.[20] ('Formal' as applied to logic has other

[18] Recall that Sher actually speaks of 'invariance under *isomorphisms*', though as mentioned in note 3 above, the further structure added to a domain is essentially an idle wheel of her theory.

[19] We saw in an earlier note that this can actually be questioned, at least for some extensional constants that appear to have some degree of logicality, such as 'is a part of' and 'is true'. But it may well be that all extensional *clearly* logical constants are invariant under bijections. We will say a bit about this in section 4 below.

[20] As Gil Sagi points out to me in p.c., Sher evidently sees as a virtue of her account that it characterizes logical constants in terms of properties of their contents, the logical operators (that's after all why she has a semantic conception of logic), much as the properties of the contents of the terms peculiar to biology, mineralogy, etc., presumably characterize those sets of terms. Sher also evidently sees as a virtue of her account that the property that characterizes logical operators on her account is one that has some independent right to get the Protean word 'formal' applied to it. However, as I see things, this doesn't give us a distinctive explanation of anything we wanted explained in advance. As pointed out in the text, in the

meanings as well, which are obviously not in Sher's mind. People have spoken of formal as opposed to informal logic, broadly in the sense of mathematical vs. non-mathematical logic; and of formal vs. material logic, approximately to distinguish the science of correct reasoning from the science of sound reasoning (reasoning from true premises).)

According to Sher (see (2008), 305ff.), her identification of logicality or formality with invariance explains the generality and topic-neutrality of logic, because bijections are highly unrestricted transformations compared to other transformations. Thus she recalls Tarski's (1966) comparison of permutations of a domain of geometric points with other transformations of such domains in which more relations between items in the domain have to be preserved: isometric transformations (in which distance between points is preserved), similarity transformations (in which the ratio of the distance between any two points is preserved), affine transformations (in which the mutual linear position of any three points is preserved), and continuous transformations (in which continuity in a set of arguments is preserved in the corresponding set of values). But Sher notes that there are other transformations between domains that require even less than bijections, the most unrestricted type of transformation being perhaps that of an arbitrary function; so invariance under bijections does not characterize "utmost generality" of notions. Nevertheless, insofar as bijections are a highly unrestricted type of transformations, invariance under bijections does capture generality in some appropriate sense, and indirectly topic-neutrality, even if it doesn't capture "utmost generality".

However, the level of unrestrictedness of the transformations involved in a notion of invariance has little to do with what is normally understood by the generality and the topic-neutrality of logic. The counterexamples of section 2 indicate a way of seeing this. That an expression is general means that it has wide application, in all or nearly all areas of discourse. That an expression is topic-neutral means that its topic is somehow common to all or not peculiar to any of the specific areas of discourse. Expressions such as 'unicorn', 'heptahedron' and 'male widow' (and their more complicated versions 'unicorn#', 'heptahedron#' and 'male widow#') are not general or topic-neutral in these senses, despite being invariant under bijections, so Sher's explanation of the generality and topic-neutrality of logic is of dubious value. (I take it for granted here that all these examples are invariant under bijections of arbitrary domains, on any theoretically viable way of understanding this notion, assuming that, as argued in section 2, Sher's rejection of the

relevant pre-theoretical sense of 'formal', any theory of logical constancy that is not blatantly inadequate provides some explanation of the formality of logic.

examples by appeal to domains with fictional elements and "conventionally designated" elements is based on catastrophic theoretical choices.)

She might protest that she is offering an explanation of the generality and neutrality of logical operators, not an explanation of the generality and topic-neutrality of expressions in other senses, e.g. in the sense of full-fledged expressions with all the aspects of their content settled. An operator in Sher's sense will be general and topic-neutral if it is invariant under bijections, even if many expressions in the intuitive sense, some of them intuitively non-general and non-topic-neutral, can correspond to one and the same operator. This may be an acceptable reply, but whether it is acceptable or not will depend on whether there is a reasonable intuitive sense in which operators as such can be classified as general or non-general, or topic-neutral or non-topic-neutral, and such that it corresponds approximately to their being invariant or non-invariant. Such a sense surely exists for linguistic expressions as usually understood, expressions endowed with full meaning or content—it's this meaning or content that determines their intuitive topic, for example. But it is unclear to me that the appropriate sense exists for operators as Sher understands them. After all, an operator in Sher's purely extensional sense is supposed to be defined over every domain,[21] regardless of the nature of its objects, whether it turns out to be invariant under bijections or not, and, as noted, one same operator is associated with many intuitively different topics.

Be this as it may, we will now see that Sher's explanations of the necessity, strong normativity and (quasi-)apriority of logic suffer from related problems, which in these cases certainly cannot be shrugged off by an appeal to the distinction between full-fledged linguistic expressions and operators.

Let's begin with Sher's explanation of the quasi-apriority of logic. She holds a Quinean view on which logic is not *a priori*, because some empirical discoveries might lead one to revise one's logic (she cites the suggestion of Birkhoff and von Neumann, Putnam and others that quantum-mechanical experiments provide an empirical justification for a change in logic). However, logic is "quasi-a-priori",

[21] It may be worth noting, however, that on Sher's own standards it is unclear that her operators are well defined, not just in view of the vagueness of the terms with the help of which they are defined in the metalanguage, but because, as we saw, Sher claims that vast numbers of terms from natural language are undefined for many domains; and it is just impossible to get out of natural language if we want to define metalinguistically the operator for 'red', say.

> in the sense that the logical laws themselves are unlikely to be refuted by our empirical discoveries. The Invariance-under-Isomorphism criterion explains why logic is immune to refutation in this sense. Since most of our empirical discoveries do not concern the formal regularities in the behavior of objects—i.e., regularities governing features of objects that are invariant under isomorphism—logic is not affected by most of these discoveries, and in this sense it is resistant to refutation and, furthermore, a-priori-like. (Sher (2008), 317)

Even granting for the sake of argument that logic is not *a priori* but quasi-a priori in some sense, one problem with this alleged explanation is that not all "regularities governing features of objects that are invariant under isomorphism" need be "unlikely to be refuted by our empirical discoveries". Many "features of objects" which are invariant under bijections will appear in the statement of "regularities" which need not be particularly resistant to undermining by new empirical information. Take 'unicorn' (or 'unicorn#'). Suppose it is indeed the case that 'unicorn' has the empty extension in all (metaphysically possible) domains. It then corresponds to a "feature of objects" which is invariant under bijections, and which appears in the "regularity" '∀x~unicorn(x)'. But this doesn't mean that it must then be unlikely that new empirical discoveries will undermine our belief in this regularity. If skeletons of equine creatures with something that looks like a horn on their heads are discovered around a place where it is believed that (the earliest ancestor of) the word 'unicorn' originated, our belief that there were never any unicorns would be undermined (even if the belief is ultimately correct). I don't know how likely or unlikely it is that such a discovery will be made, but the important point is in any case that the invariance under bijections of 'unicorn' clearly doesn't guarantee that the discovery is unlikely.[22] (Of course, much less could it guarantee the full apriority of '∀x~unicorn(x)", for this, or its attitudinal content, is evidently *a posteriori*.)

Sher might again reply that she is offering an explanation of the (quasi-)apriority of regularities involving logical operators, not an explanation of the (quasi-)apriority of regularities involving full-fledged expressions or their contents. But now the reply is evidently not adequate, for a "regularity", if it is to be something of which it makes sense

[22] For another, perhaps clearer example, consider the predicate 'coinpockeight', which means "coin that is inside Mario Gómez-Torrente's pockets on March 8, 2017". 'Coinpockeight' would seem to be a clearly non-logical constant if the distinction logical/non-logical is a substantive one. But it turns out that its extension in all universes of actual existents is the empty set. 'Coinpockeight' corresponds to a "feature of objects" which is invariant under bijections, and which appears in the "regularity" '∀x~coinpockeight(x)'. But this doesn't mean that it must then be unlikely that empirical discoveries will undermine a belief in this regularity. It may well be quite likely that people will discover plenty of evidence suggesting that I must have carried some coins on that day after all, such as that I have always been observed carrying coins, that I used some coins that day to get a drink from a vending machine, etc.

to say that it is (quasi-)a priori or (quasi-)a posteriori, has to be something of which it makes sense to say that it is susceptible of being the object of propositional attitudes such as knowledge and (justified) belief. An operator in Sher's sense is pretty clearly not something that can be the object of propositional attitudes, or at least can only be such an object through the mediation of some corresponding attitudinal content, a kind of thing that is normally associated with the idea of a full-fledged linguistic content. A "regularity" involving merely an operator in Sher's sense, whatever such a regularity might be, is thus not something that could be justified or undermined as a part of a belief system, whether by empirical or non-empirical information. Only a regularity involving something essentially analogous to a full-fledged content, the content of a full-fledged linguistic expression, is susceptible of being *a priori* or *a posteriori*, and this renders Sher's attempted explanation of the (quasi-)apriority of logic vulnerable to our objection above.

A related problem affects Sher's purported explanation of the "basicness" and strong normative force of logic. The phenomenon to be explained is that logic appears to hold some kind of overarching authority over other less "basic" disciplines.

> Chemistry, biology, and geography have to attend to the strictures of logic, but logic need not attend to their strictures. Logic has normative authority over these disciplines, but not vice versa. The Invariance-under-Isomorphism criterion explains why this is so: Since chemical properties are not preserved under isomorphisms, logic has a stronger invariance property than chemistry. As a result, logic does not distinguish chemical differences between objects and is not subject to the laws governing chemical properties. But chemistry does distinguish formal differences between objects; for example, it distinguishes between one atom and two atoms. So chemistry is subject to the laws of formal structure. (Sher (2008), 317)

Now, here again "laws of formal structure", if they are to be something that thinkers must abide by, have to be things susceptible of being accepted or rejected by them. "Laws" involving merely operators in Sher's sense are not things that can be obeyed or disobeyed as such; only laws involving full-fledged contents can be obeyed or disobeyed. And then the problem is that many laws involving full-fledged contents that determine operators invariant under bijections are nevertheless intuitively not "basic" or authoritative over non-logical disciplines in general. There is no intuitive sense in which '∀x~heptahedron(x)' (or '∀x~heptahedron#(x)') is authoritative over arithmetic or geometry, and there is similarly no sense in which '∀x~male widow(x)' (or '∀x~male widow#(x)') is authoritative over physics or biology, even though 'heptahedron' (and 'heptahedron#') and 'male widow' (and 'male widow#') correspond to operators which are invariant under bijections.

The situation is somewhat different in the case of Sher's explanation of the necessity of logic. This is condensed in the following passage:

> The totality of models represents the totality of formal possibilities; logical consequences preserve truth across all models; they do so due to the logical structure of the sentences involved; this logical structure reflects the formal skeleton of the situations described by those sentences; therefore the preservation of truth is due to connections that hold between the formal skeletons of the situations involved in all formal possibilities; and formal connections persisting through the totality of formal possibilities are laws of formal structure. It follows that consequences satisfying Tarski's definition are formal and necessary, as required by the intuitive constraints (however strong the necessity constraint is taken to be). (Sher (2008), 316)

While Sher's presentation is perhaps less than fully clear, the relevant train of thought seems to be reconstructable as follows. (Compare the related argument in McGee (1992) and its generalization in Gómez-Torrente (2000), ch. 8.) (i) That a sentence is a Tarskian logical truth means that it is true in all its (actual set-theoretical) models (interpretations of its non-logical constants); similarly, that an argument is a Tarskian logically correct argument means that it is truth-preserving in all its (actual set-theoretical) models (interpretations of its non-logical constants). (ii) That a sentence is necessary means that it is true in all possible worlds; similarly, that an argument is necessary means that it is truth-preserving in all possible worlds. (iii) Suppose that a sentence S is not necessary and an argument $P_1,...,P_k/C$ is not necessary. Then S is false in some possible world *w* that provides an interpretation for its non-logical constants, and $P_1,...,P_k/C$ is non-truth-preserving in some possible world *w'* that provides an interpretation for its non-logical constants. (iv) But then, since items in logical form do not distinguish between bijectable domains, S will be false in every isomorphic interpretation in *w* (or in any other world), and $P_1,...,P_k/C$ will be non-truth-preserving in every isomorphic interpretation in *w'* (or in any other world). (v) Models consisting just of pure sets include models of all the forms that can be drawn from all the different possible cardinalities, so S will be false in a model consisting just of pure sets in *w*, and $P_1,...,P_k/C$ will be non-truth-preserving in a model consisting just of pure sets in *w'*. (vi) But these pure sets models are also actual set-theoretical models, so S will be false in some actual set-theoretical model and $P_1,...,P_k/C$ will be non-truth-preserving in some actual set-theoretical model. (vii) Therefore, logical truths and logically correct arguments are necessary.

Now I agree that, with some probably surmountable caveats,[23] this is a valid derivation of the necessity of sentences and arguments from the assumptions that they are

[23] For example, (i) is plausible for higher-order sentences only if we assume that all non-set-theoretical interpretations of them are somehow adequately represented by models, which are normally assumed (and assumed by Sher) to be set-theoretical; this presupposes the truth of otherwise plausible reflection principles about sets. For another example, (vi) arguably presupposes, again plausibly in my view, that the

Tarski-valid and that their logical constants are invariant under bijections in *arbitrary possible* worlds. I stress 'arbitrary possible' both because invariance under bijections in arbitrary possible worlds and not merely invariance under bijections of actual domains is needed in the derivation, and because the fact that invariance under bijections in arbitrary possible worlds is what is needed deprives the derivation of much value as an explanation of necessity in terms of invariance. Let me explain these two points.

That an expression is invariant under bijections in arbitrary possible worlds, or necessarily invariant under bijections, means simply that given an arbitrary world, any two domains of things existing in that world, and a bijection between these domains, the extension of the expression in the second domain is the image under the bijection of its extension in the first domain. Intuitively, all standard logical constants of classical quantificational languages are necessarily invariant under bijections. But that logical constants are necessarily invariant under bijections is evidently required in step (iv) of the derivation above. If a sentence S is made false by a model existing in some other possible world, and we can only assume that it is invariant under bijections of actual domains, or even under bijections of domains restricted to some particular possible world, then we will not be able to conclude that S is made false also by a model existing in the actual world. This is the first point.

The second, more important point, arises from the first. The point, to put it in a nutshell, is that the purported explanation of necessity in terms of invariance is not really such, insofar as the derivation, however valid, deduces the desired modal property from a closely related modal property of the relevant expressions, not from invariance or formality by itself. To take a simple case, consider the predicate '∅' above, defined to be synonymous with 'non-identical to itself'. The sentence '$\forall x{\sim}\varnothing(x)$' is intuitively a Tarskian logical truth, as '∅' is intuitively a logical constant and, under the assumption that it (and '∀' and '~') is a logical constant, '$\forall x{\sim}\varnothing(x)$' is true in all actual set-theoretical models. And '$\forall x{\sim}\varnothing(x)$' is also intuitively necessary, e.g. in the sense that intuitively no object from any domain in any possible world is non-identical to itself. However, that '$\forall x{\sim}\varnothing(x)$' is necessary doesn't follow from the facts that '∅' is invariant under bijections of actual domains and that '$\forall x{\sim}\varnothing(x)$' is true in all actual set-theoretical models; it only follows from the assumption that '∅' is invariant under bijections of domains in arbitrary possible

universe of pure sets is strongly uniform from world to world, i.e. that it's not richer in some worlds than in others (a version of the traditional view that mathematical objects are necessary existents).

worlds. (The reasoning is again this: if '∀x~∅(x)' were false in some possible world, it would be false in a domain from that world, hence false in a domain of pure sets—this is the step that uses invariance under bijections in arbitrary possible worlds—hence false in some actual model.) But the assumption that '∅' is invariant under bijections of domains in arbitrary possible worlds is just too short a distance apart from the claim that '∅' has the empty extension in all domains irrespective of the possible world they come from. And this is just too short a distance apart from the very claim that '∀x~∅(x)' is necessary. Sher's derivation of the necessity of Tarskian logical truths surreptitiously uses intuitive modal properties of expressions in order to derive closely related modal properties of the sentences they appear in; but this is no real explanation of those modal properties, let alone one in terms of invariance or formality by itself. A real explanation must provide a substantial account of what it is that determines that '∅' is necessarily empty, and thus invariant under bijections of domains in arbitrary possible worlds.[24]

Now while Sher ascribes primary importance to the arguments that her proposal is explanatory of several philosophically important characteristics of logical truths and logically correct arguments, she also sees strong considerations in favor of her view in claims that the proposal is theoretically rich in a number of other ways:

> …the Invariance-under-Isomorphism criterion (…) has opened new areas of research in mathematics and linguistics and helped solve standing problems in both disciplines. [footnote:] For example, it has led to the development of "model-theoretic logic" and "generalized quantifier theory." Some remarkable results of these new fields are Lindström's characterization of (standard) first-order logic, Keisler's completeness proof for first-order logic with the quantifier "uncountably many," the solution to the problem of determiners in linguistic semantics, and the theories of polyadic and branching quantifiers in natural language. (Sher (2008), 308, and n. 11 *ibid.*)

But I suspect that there is at least a good amount of overstatement here. It's unclear to me that the "Invariance-under-Isomorphism criterion", by which Sher means her *characterization* of logicality, the statement that invariance under bijections is necessary and

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[24] For another example reinforcing this point, consider again the predicate 'unicorn'. The necessity of '∀x~unicorn(x)' is surely a consequence of the facts that 'unicorn' (and '∀' and '~') is invariant under bijections of arbitrary possible domains and that '∀x~unicorn(x)' is true in all its models when one takes invariance under bijections as one's criterion of logicality. But this doesn't mean that these facts provide a real explanation of the necessity of '∀x~unicorn(x)'; a real explanation must involve mention of aspects of the meaning and/or the mechanism of reference fixing of 'unicorn' that determine what its extension is in a possible world.

I record my debt to Gil Sagi for pushing me to clarify my critique of Sher's explanation of the necessity of logical truth.

sufficient for the logicality of an expression (or operator) has *per se* "opened new areas of research in mathematics and linguistics and helped solve standing problems in both disciplines". The *notion* of invariance under isomorphisms or bijections may have played such a role, but it seems doubtful to me that a view of *logicality* has.[25]

As I see things, for example, research in model-theoretic logics is certainly based on a definition of an abstract logic which incorporates the condition that a sentence of a logic has the same truth value in all isomorphic structures. But this doesn't presuppose any view of the logicality of an expression, and it is in fact compatible with treating some expressions which are invariant under bijections as non-logical; the notion of a non-logical constant involved in these researches is a merely stipulative one that regards all predicate, function, and individual symbols (aside from identity) as non-logical regardless of their meanings (see note 3 above). If anything, these researches presuppose at most that invariance is a necessary property of logical expressions. Similarly, generalized quantifier theory does make use of the notion of invariance under bijections or invariance under permutations in some of its tentative generalizations, but as far as I can tell it does not presuppose any view of logicality (even if some authors may stipulatively use 'logical' as synonymous with 'invariant'). Thus, for example, in their standard exposition of generalized quantifier theory Peters and Westerståhl (2006) are at pains to argue that there is no non-arbitrary view of logicality, and in particular that invariance under bijections provides at best a necessary condition of logical quantifiers and other logical expressions (see especially Peters and Westerståhl (2006), ch. 9).

In any case, it's unlikely that arguments based on the richness of technical work sparked by a notion can provide any very substantive ground on which to base a choice of one theory of logicality over others, and in particular over other theories that Sher evidently wants to claim are inferior to hers. (Though I don't mean to imply that Sher lends

[25] In this connection it's worth remarking that the situation with logical constancy is significantly different in one important respect from that with other notions for which theoretical reconstructions have been offered in the modern logical tradition, such as truth, truth in a model, validity and computability. The reconstructions of these notions were always attempted at least to a good extent because the notions in question were *already* involved in technical results produced by logicians and mathematicians, which fact created a pressure for rigorous versions of the notions within the framework of accepted mathematics. It's to say the least unclear that anything similar can be said of the notion of logical constancy. (To be sure, Tarski's proposal of his own theory of logical consequence relied on the notion and thus generated a desire to reconstruct it as well, but it's unclear that the notion is relied upon in any earlier "natural" technical result.)

so much weight to this kind of arguments.) The invariantist notions of Feferman and Bonnay have led and can lead to fruitful technical research too, and so have other notions used in attempts to characterize logicality, of the inferentialist kind, for example. Furthermore, and to my mind more importantly, if we were to use technical theoretical richness as a criterion with which to adjudicate the dispute at stake here, I think that would load the dice inappropriately in favor of mathematical notions, which will by their very nature tend to be fruitful technically even if they can provide only mediocre extensional approximations of the notion of logicality. As I have argued in earlier work and will argue again in section 4, my suspicion is that the intuitive idea of a logical constant involves many notions which are predominantly pragmatic in nature, and which are likely to resist approximation via mathematical and other more or less "technical" tools, and to use technical theoretical richness as a standard will just not be very fair to views of this kind.

It is now time to discuss such views, and to see if there are reasonable hopes that they could provide a more satisfactory account of logicality than invariantist views, even from the point of view of the semantic tradition.

4. The semantic conception of logic and the proper outlook on logical constancy

Invariantist theories are not alone among theories of logicality in facing serious objections. In my earlier work, I have argued that there appear to be difficulties of principle for all theories of logical constancy that are formulated purely in terms of mathematical or semantic notions. I would like to stress that this is a weaker view than the view of some detractors of the semantic conception of logic, who have argued against the possibility of a non-arbitrary divide between logical and non-logical constants as a part of their case against Tarski's method for defining logical consequence (see e.g. Etchemendy (1990), ch. 9; Read (1994)). It's also important to note that, if I am right and an adequate semantic or mathematical theory of logical constancy were in fact impossible, this would not really be a serious problem for the semantic conception of logic. Tarski would have undoubtedly welcomed a substantive mathematical explication of the notion of logical constancy that could have permitted its elimination from the general statement of his method for constructing definitions of logical consequence for particular languages. However, he himself noted that his method could get by without such a general explication, defining 'logical constant' by enumeration for particular languages (see Tarski (1944)). Furthermore, when he presented his idea of defining 'logical notion' in terms of invariance under permutations in (1966), he made it clear that in his view the idea did not settle once and for all questions as to the scope of logicality, and that what concepts counted as logical in a language still depended on arbitrary decisions of some kinds. This was the latent thought in Tarski all along, especially in the seminal paper on logical consequence, where he says that "no objective grounds are known to me which permit us to draw a sharp boundary between the two groups of terms [logical and extra-logical]" (Tarski (1936), 418-9), and that

> I also consider it to be quite possible that investigations will bring no positive results in this direction, so that we shall be compelled to regard such concepts as 'logical consequence', 'analytical statement', and 'tautology' as relative concepts which must, on each occasion, be related to a definite, although in greater or less degree arbitrary, division of terms into logical and extra-logical. (Tarski (1936), 420)

Clearly Tarski did not think that the value or the correctness of his method for defining logical consequence would have been threatened by this possibility.

My positive view on the question of the characterizability of the notion of logical constant is, in fact, more optimistic than Tarski's latent view or his detractors' explicit view. I think that a descriptively adequate characterization of logical constancy may well be possible if it is given in terms of properties which, unlike those involved in the usual characterizations, involve reference to certain interests of human beings as argumentative and truth-seeking creatures. While I cannot propose a characterization or list of necessary and sufficient conditions for logical constancy with which I am completely satisfied, I think I can give some informative indications on the form that an appropriate characterization would take. These indications make it clear already that, if a characterization of this form is correct, then the distinction between logical and non-logical constants is not arbitrary. Besides, the distinction that would emerge from such a characterization would be descriptively adequate in all the ways in which invariantist and other characterizations are not. Furthermore, the indications I will give will also suggest how a characterization of this form might be coupled with a certain kind of account of how it is that logical truths and logically correct arguments have the philosophically important properties they are traditionally thought to have. And finally, the indications also suggest an explanation for the recurring difficulties of usual characterizations.

Much of the *raison d'être* for logic is the need to isolate and study relatively uncontroversial, theory- and topic-neutral bases for argumentation and reasoning, or at least for argumentation and reasoning that is in some sense important or relevant theoretically or practically. Parties in a discussion or in the evaluation of claims of any sort need a common ground that is not under dispute or under examination. Such a ground need not always be logical in nature, but logic arises as the study of a special need arising from this general need; its aim is to identify important sentential forms and forms of argument whose instances are not just indisputable in particular contexts, but in all contexts or as many contexts as possible, and specifically independently as much as possible of the nature of the claims or of the topic which is being discussed or evaluated.[26]

These motivations are vague and some are pragmatic. It is thus natural to conjecture that the notion of a logical constant is correspondingly vague along various dimensions, some of them pragmatic, which parallel these motivations. To begin with, logical constants must be words which are generally applicable regardless of the theory or the topic under discussion. But 'generally applicable' is vague, there being no clear-cut line separating the generally applicable linguistic expressions from the non-generally applicable. The right view would seem to be that there is a dimension of degrees of general applicability on which expressions are ranked, with the ranking influencing the degree of logicality that an expression is perceived to have.

Expressions such as 'is a part of', 'is true' and others are generally applicable to a great degree, but they are logical in some degree that appears intuitively inferior to the degree in which expressions such as 'not', 'and' and 'there are things such that' are logical. At least one philosopher sympathizing with a pragmatic outlook (Warmbrōd (1999)) has suggested that logical constants must satisfy a necessary condition stricter than general applicability: they must appear necessarily in the systematization of deductive scientific reasoning. Perhaps expressions such as 'is a part of' and 'is true' fail to meet this requirement, but the requirement seems too strict anyway. First of all, it doesn't motivate appropriately the restriction to scientific reasoning; surely other kinds of reasoning might be kinds of reasoning that logic (even logic as traditionally conceived) would aspire to systematize. Second, it leaves unexplained the fact that expressions such as 'is a part of' and 'is true' appear to have at least some degree of logicality. Third, read literally, the requirement might leave out all expressions altogether, for perhaps no expression in particular is really necessary for the systematization of deductive scientific reasoning;

[26] This is true at least of logic as traditionally conceived and understood. Surely the concept of logic has undergone changes through time, and especially over the last fifty years or so, when the feasibility of logic as traditionally conceived has been called into question and the word 'logic' has become applicable to sets of sentential forms and forms of argument whose validity is restricted to highly particular contexts or topics. I assume, with most discussions of logical constancy (including Sher's), that the traditional logical ideal is not unrealizable, and that it has been realized, however partially, with the identification and description of things such as propositional logic and first-order logic.

perhaps every particular expression can be appropriately replaced without this affecting the relevant qualities of the systematization.

Warmbrōd (1999) has also proposed that his condition of appearing necessarily in the systematization of deductive scientific reasoning is merely a necessary condition on logical constancy, and has rejected the project of characterizing the concept in terms of necessary and sufficient conditions. While I think it may be wise in general to be skeptical about the possibility of characterizations of pre-theoretical concepts in terms of jointly necessary and sufficient conditions, in the special case of logical constancy I don't see compelling reasons to give up the idea of a characterization. In particular, I don't think Warmbrōd provides any discernible reason why we should think that if his postulated requirement were correct, it should not be, besides necessary, also sufficient for logical constancy.

The most natural hypothesis, absent some strong specific motivation to the contrary, is that the other intuitive dimensions corresponding to the vague motivations for the singling out and study of logically valid sentences and arguments play the required role here. First, in rounding out the basic intuition of general applicability into a characterization of logicality in terms of (vague) necessary and sufficient conditions, and second, in explaining the intuitively lower degree of logicality of expressions such as 'is a part of' and 'is true'.

As noted above, the intuitions that drive the singling out of expressions for logical study implicitly require that logical constants be important or relevant theoretically or practically (thus in scientific reasoning in particular, but not exclusively), where this importance or relevance may be measured in several ways, e.g. by the degree of usefulness of paying attention to the expression in the resolution of significant problems or disputes in reasoning in general. Again it would seem that there is a dimension of degrees of importance or relevance on which expressions are ranked, with the ranking influencing the degree of logicality that an expression is perceived to have. This reasonably introduces one kind of vagueness that might assign expressions such as 'is a part of' and 'is true' a lesser degree of logicality than that of paradigmatic logical constants, even if their degree of general applicability is comparable.

A third dimension of variation corresponding to one of the intuitive motivations for logic as a distinguished field of study is the dimension of degrees of controversiality of the principles apparently governing a linguistic expression.[27] The higher the degree of

[27] Here, as in many other places, and along with nearly every other author in the literature on logical constants (including Sher), I am assuming that our pre-theoretical views about logicality concern expressions existing

uncontroversiality of the principles apparently governing an expression the higher its perceived degree of logicality could be expected to be.

In my view, it is not implausible to think that these dimensions, along with perhaps other related ones, play a constitutive role in the pre-theoretical concept of a logical constant and determine its (vague) extension in terms of (vague) necessary and sufficient conditions. A logical constant is, I conjecture, an expression that ranks high in all the relevant dimensions. A clearly logical constant must then rank very high in all dimensions, and a clearly non-logical constant will rank very low in at least some of the dimensions.

The vagueness of the ensuing concept of logical constancy is then in direct correspondence with the vagueness of concepts such as those of general applicability, importance and relevance to general reasoning, and uncontroversiality. Some of these concepts, furthermore, are pragmatic, in the sense that they involve notions that make implicit reference to relations between expressions and the practical interests of human beings, such as importance, relevance, and uncontroversiality. But, as advanced above, the vagueness and the pragmatic nature of the postulated concept of a logical constant provide several things the theorist of logical constancy wants: a refutation of the attempts to dissolve the problem of logical constants; an account that is arguably descriptively correct *vis-à-vis* the examples that have been considered in the literature; a certain account of how it is that logical truths and logically correct arguments come to have the philosophically important properties they are traditionally thought to have; and an explanation of the failure of the usual attempts to solve the problem, and in particular of the failure of invariantist proposals.

In the first place, the vagueness of the characterization refutes the mentioned attempts to dissolve the problem, which postulate that the distinction between logical and non-logical constants has no real difference behind. Why is this? What vagueness does is to leave a large borderline area of expressions that don't clearly rank high on (some of) the relevant dimensions but also don't clearly fail to rank high. Are these constants in the borderline area of (some of) these vague dimensions logical constants? The arbitrariness theorist is right that in these cases there is no clear answer. The vagueness of the concept is compatible with many incompatible ideas about what expressions can be considered logical. It's therefore true that this vagueness implies that we cannot suppose without

in natural language, and thus that the principles governing these expressions are not given by stipulation (as one might argue is the case for at least some of the "symbolic" correlates of logical expressions in artificial languages). Hence the possibility that these principles will be controversial, and in various degrees.

further ado that every argument has its logical form. If an argument contains expressions which are neither clearly logical nor clearly non-logical, then we cannot speak of a single logical form for that argument. This has motivated in part the attempts to dissolve the problem of logical constants by claiming that the question of which expressions are logical is arbitrary. But the fact that the question of which expressions are logical is vague doesn't imply that it is arbitrary, because the dimensions involved in the vague concept of logical constancy are not compatible with just any idea about which constants can be considered logical. The vagueness of a concept does not imply that its application is arbitrary, and in fact it excludes this, as it is based on the existence of clear cases of application and clear cases of non-application. What is arbitrary to some degree is which constants of the borderline area to consider as logical or non-logical. But if an argument contains only words which are either clearly logical or clearly non-logical, then the logical form of that argument is fully determined by the intuitive conditions on logical constants. So our characterization makes it clear that the distinction between logical and non-logical constants is not arbitrary.

In the second place, our characterization is also descriptively adequate, at least insofar as the examples we have considered are concerned. Expressions such as 'not', 'and', '=', 'is non-self-identical' and 'there are things such that' rank high in all the identified dimensions and are thus presumably logical on any appropriate completion of the proposal. Expressions such as 'is a part of' and 'is true', and also many expressions with a peculiarly set-theoretical content, such as 'belongs' (in the set-theoretical sense), 'set', 'function', 'there are infinitely many things such that' and 'there are uncountably many things such that' don't seem to rank as high, at least in some of the dimensions. For example, undoubtedly all these constants have a somewhat less general applicability than the clearly logical constants. They appear also to be less relevant in reasoning in general, and their study appears less useful for the solution of problems about the validity of arguments in general than the study of the paradigmatic logical constants. And finally, the principles governing them appear to be more controversial than the principles governing these constants. (Note then that our characterization provides an explanation for the fact that (contra Sher), the question about where to locate the border between logic and mathematics does seem to be vague. This is explained by the vagueness of the dimensions relevant to logical constancy: plausibly, the more intuitive mathematical content a constant has, the farther away it is from the clear cases of logical constancy along (some of) these dimensions.[28]) On the other hand, 'unicorn', 'heptahedron', 'male widow' and their '#' versions presumably rank pretty low in all or most dimensions (perhaps some of them don't rank low in the dimension of uncontroversiality), and are thus appropriately declared (clearly) non-logical by our characterization.

In the third place, our characterization provides accounts of how it is that logical truths and logically correct arguments come to have at least some of the philosophically important properties they are traditionally thought to have. The generality and topic-neutrality of logic is automatically guaranteed by the characterization, insofar as this requires that logical constants, the constants responsible for the logical validity of truths and arguments, rank high in the dimension of general applicability. And it's worth noting that generality and topic-neutrality presumably explain that (many or all) clear logical constants are invariant under bijections (not the other way round, as in Sher). That a constant is invariant under bijections follows from the requirement that a clearly logical constant must be generally applicable to a very high degree, under the assumption that a constant generally applicable to a very high degree must not distinguish between bijectable extensions (between individuals, in the simplest case).[29]

The basicness and strong normativity of logic is presumably a consequence of its generality and topic-neutrality coupled with the requirement of high uncontroversiality. For high uncontroversiality guarantees the epistemic strength that is required of a principle if it is to hold the needed authority in discussions and evaluations of material from non-logical disciplines. But generality and topic-neutrality are also needed, for otherwise the authority

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[28] The same sort of observation may go a long way toward providing also an explanation for the feeling, emphasized by Bonnay ((2014), 62f.) and Feferman ((2010), 17), that the choice between the several invariantist notions in the literature on logical constancy may be arbitrary to a good extent.

[29] It may be worth noting that I see interesting connections between John Burgess's (2015) recent suggestions concerning structuralism and the present proposal. According to Burgess, the source of the apparent prevalence of a structuralist outlook on the part of mathematicians is a byproduct of their search for rigor: as soon as one sets out to derive one's theorems from basic principles about certain notions, these principles and the theorems derived from them will also hold for any structure isomorphic to one in which the principles hold, in virtue of the fact that the usual logical constants are invariant under isomorphisms. (And this will be so independently of whether some particular structure was the one that mathematicians originally intended to describe.) While I suspect Burgess's suggestion is right, it leaves unexplained the very fact that the usual logical constants are invariant under isomorphisms. On the present view, this is again a byproduct of a more fundamental fact, one that, like the search for rigor, answers to pragmatic features of a certain human endeavor: the interest in isolating expressions that can serve as the basis of logical reasoning because they are, among other things, generally applicable and topic-neutral to a very high degree.

in question would not be overarching. On the other hand, generality, topic-neutrality and uncontroversiality seem to provide intuitively jointly sufficient conditions for strong normativity in the sense that concerns us.

Let's now focus on apriority and necessity. Unlike Sher's invariantist proposal, the account I have in mind does not seek to derive the apriority and necessity of logical truths and logically correct arguments from the features of the proposed notion of logical constancy. In my view, it is doubtful that the explanation of the apriority or necessity of logical truths and logically correct arguments can be a peculiar explanation that relies on the peculiarities of a group of expressions. Apriority and necessity presumably have homogeneous sources across different kinds of sentences and arguments. The explanation I have in mind takes it for granted that such an explanation must exist, whatever it is, both for logical truths and logically correct arguments and for other kinds of *a priori* and necessary truths and arguments, and instead suggests an account of why some of those truths and correct arguments are selected as special concerns of logic.

The account is based on the idea that logic seeks to serve the need for a special study of the (largely) uncontroversial bases for argumentation and reasoning in all contexts. At the level of the expressions it studies, this has the effect that logic seeks to study expressions governed by principles which are as safe as possible; this is reflected in the condition on logical constants that the principles governing them should be uncontroversial to a large degree. But it also has another effect, at the level of the sentences and arguments: it has the effect that the logical truths and the logically correct arguments must be uncontroversial to a large degree, since they are true or correct in virtue of the principles governing the non-schematic expressions in their logical forms. Indirectly, this ultimately determines that logical truths and logically correct arguments are *a priori* and necessary, as uncontroversial truths and forms of arguments valid for all contexts or as many contexts as possible will have those properties. Note, for example, that if logic is to provide a largely uncontroversial ground for reasoning in all contexts, it must do so for arithmetical reasoning; and in this context the uncontroversial ground for reasoning cannot be constituted by contingent propositions or principles or by *a posteriori* propositions or principles. A reasoning basis which included contingent or *a posteriori* propositions or principles would not be as strong a basis for arithmetic as reasoners would desire one such basis to be.

In the fourth place, the pragmatic nature of at least some of the intuitive dimensions involved in the concept of logical constancy explains to a good extent the descriptive problems of the traditional attempts to characterize them. The difficulties are due to the fact that logicians and philosophers of logic have almost always tried to offer their characterizations exclusively in terms of mathematical properties, invariantist proposals being a prime example of this phenomenon. It is very implausible that the pragmatic intuitions underlying the intuitive concept of a logical constant can be captured via a search for mathematical peculiarities. I can't show, of course, that it is impossible to offer a good characterization of logical constants (that is, one that contains in its extension the clear positive cases and leaves out the clear negative cases) exclusively in terms of mathematical properties. But I think it's obvious that it must be extremely difficult to characterize a notion that makes reference to certain practical interests of human beings in terms of properties of this kind, and I think that the counterexamples of section 2 bear witness to the difficulties.[30] (It may be noted that other concepts that the recent logical tradition has characterized relatively successfully (such as the concepts of truth, validity (with respect to the standard quantificational constants), computable function, etc.) seem to be, already at an intuitive level, fully objective concepts lacking *a priori* connections with concepts involving the practical interests of human beings.)

It is of course to be expected that some conditions definable exclusively in terms of mathematical properties be necessary conditions of the intuitive logical constants, or of significant groups of them, and this is in fact the case. We have argued, in fact, that invariance under bijections is presumably a property of clearly logical constants that follows from their high degree of general applicability and topic-neutrality. (On the other hand, 'is a part of' and 'is true', while widely applicable, and possessing an intuitively high degree of logicality, are not invariant under bijections.) But it is not to be expected that this mathematical property should be a sufficient condition for logical constancy in the intuitive sense, because it is not to be expected that all the constants with that mathematical property can be generally applicable (or that they should all satisfy the other intuitive pragmatic conditions on logical constants). This idea is fully confirmed by our critical appraisal of invariantist theories.

Thus, a pragmatic characterization of the kind we are envisioning promises a founded response to the attempts to dissolve the problem of logical constants, an adequate approach to descriptive problems, a certain account of how it is that logical truths and logically correct arguments have some of the philosophically important properties they come to have, and an explanation of the vulnerability of other characterizations to

[30] I must insist, however, that 'extremely difficult' does not mean impossible.

objections such as the ones developed in section 2. It remains to be seen whether a pragmatic characterization of this kind is liable to weighty objections of other kinds. It's not easy to imagine clear counterexamples to its descriptive adequacy, due precisely to its vagueness and its pragmatic character. Perhaps some critics will want to claim that its vagueness, or its pragmatic nature, or its non-mathematical nature, make it unacceptable to the semantic tradition in logic. But this would be confused. The reconstructions of concepts in the semantic tradition use vague and non-mathematical notions: all the Tarskian semantic definitions use the general notion of an expression and notions of particular expressions; and Tarskian definitions of satisfaction and truth for object languages containing vague and non-mathematical expressions use those same expressions in the appropriate metalanguages. Furthermore, a vague and pragmatic notion of logical constancy has, as I hope to have shown, intrinsic explanatory virtues that can be appropriated by the semantic conception. (Here it's good to recall how Tarski himself, when considering the implications of the possibility that the distinction between logical and non-logical constants is arbitrary, suggested that this might have the explanatory virtue of accounting for what he perceived as "the fluctuation in the common usage of the concept of consequence" (Tarski (1936), 420).) In any case, and finally: if, as noted at the beginning of this section, the value and the correctness of the semantic conception of logical truth and logical consequence are not threatened by the possibility that the distinction between logical and non-logical constants is arbitrary, much less could they be threatened by the possibility that the distinction is vague and pragmatic. I conclude, therefore, that a pragmatic characterization such as the one sketched here has the potential of providing a descriptively adequate and explanatorily rich account of logical constancy within the Tarskian semantic conception of logic.

References

Bonnay, D. (2008), "Logicality and Invariance", *Bulletin of Symbolic Logic*, vol. 14, 29–68.

Bonnay, D. (2014), "Logical Constants, or How to Use Invariance in Order to Complete the Explication of Logical Consequence", *Philosophy Compass*, vol. 9, 54–65.

Burgess, J. P. (2015), *Rigor and Structure*, New York: Oxford UP.

Casanovas, E. (2007), "Logical Operations and Invariance", *Journal of Philosophical Logic*, vol. 36, 33–60.

Cook, R. (2012), "The T-Schema Is not a Logical Truth", *Analysis*, vol. 72, 231–239.

Cook, R. (forthcoming), "There Might Be a Paradox of Logical Validity After All", forthcoming.

Etchemendy, J. (1990), *The Concept of Logical Consequence*, Cambridge (Mass.): Harvard UP.

Feferman, S. (1999), "Logic, Logics, and Logicism", *Notre Dame Journal of Formal Logic*, vol. 40, 31–54.

Feferman, S. (2010), "Set-Theoretical Invariance Criteria for Logicality", *Notre Dame Journal of Formal Logic*, vol. 51, 3–20.

Feferman, S. (2015), "Which Quantifiers Are Logical? A Combined Semantical and Inferential Criterion", in A. Torza (ed.), *Quantifiers, Quantifiers, and Quantifiers: Themes in Logic, Metaphysics, and Language*, Dordrecht: Springer, 19–30.

Gómez-Torrente, M. (2000), *Forma y Modalidad. Una Introducción al Concepto de Consecuencia Lógica*, Buenos Aires: Eudeba.

Gómez-Torrente, M. (2002), "The Problem of Logical Constants", *Bulletin of Symbolic Logic*, vol. 8, 1–37.

Gómez-Torrente, M. (2003), "The 'Must' and the 'Heptahedron'. Remarks on Remarks", *Theoria*, vol. 18, 199–206.

Griffiths, O. and A. C. Paseau (2016), "Isomorphism Invariance and Overgeneration", *Bulletin of Symbolic Logic*, vol. 22, 482–503.

Hanson, W. H. (1997), "The Concept of Logical Consequence", *Philosophical Review*, vol. 106, 365–409.

Hilbert, D. and S. Cohn-Vossen (1952), *Geometry and the Imagination*, New York: Chelsea.

Keenan, E. L. and J. Stavi (1986), "A Semantic Characterization of Natural Language Determiners", *Linguistics and Philosophy*, vol. 9, 253–326.

McCarthy, T. (1981), "The Idea of a Logical Constant", *Journal of Philosophy*, vol. 78, 499–523.

McCarthy, T. (1987), "Modality, Invariance, and Logical Truth", *Journal of Philosophical Logic*, vol. 16, 423–443.

McGee, V. (1992), "Two Problems with Tarski's Theory of Consequence", *Proceedings of the Aristotelian Society*, vol. 92, 273–292.

McGee, V. (1996), "Logical Operations", *Journal of Philosophical Logic*, vol. 25, 567–580.

Novaes, C. D. (2014), "The Undergeneration of Permutation Invariance as a Criterion for Logicality", *Erkenntnis*, vol. 79, 81–97.

Peters, S. and D. Westerståhl (2006), *Quantifiers in Language and Logic*, Oxford: Clarendon Press.

Read, S. (1994), "Formal and Material Consequence", *Journal of Philosophical Logic*, vol. 23, 247–265.

Sagi, G. (2015), "The Modal and Epistemic Arguments against the Invariance Criterion for Logical Terms", *Journal of Philosophy*, vol. 112, 159–167.

Sher, G. (1991), *The Bounds of Logic: A Generalized Viewpoint*, Cambridge (Mass.): MIT Press.

Sher, G. (2003), "A Characterization of Logical Constants *Is* Possible", *Theoria*, vol. 18, 189–198.

Sher, G. (2008), "Tarski's Thesis", in D. Patterson (ed.), *New Essays on Tarski and Philosophy*, Oxford: Oxford UP, 300–339.

Sher, G. (2013), "The Foundational Problem of Logic", *Bulletin of Symbolic Logic*, vol. 19, 145–198.

Tarski, A. (1936), "On the Concept of Logical Consequence", in Tarski, *Logic, Semantics, Metamathematics*, 2nd edn., Indianapolis: Hackett, 1983, 409–420. Translation by J. H. Woodger of "Über den Begriff der logischen Folgerung", in Actes du Congrès International de Philosophie Scientifique, fasc. 7 (Actualités Scientifiques et Industrielles, vol. 394), Paris: Hermann et Cie, 1936, 1–11.

Tarski, A. (1944), "A Philosophical Letter of Alfred Tarski", *Journal of Philosophy*, vol. 84 (1987), 28–32. A 1944 letter of Tarski to Morton White, published with a preface of the latter.

Tarski, A. (1966), "What Are Logical Notions?", *History and Philosophy of Logic*, vol. 7 (1986), 143–154. The text of a lecture originally delivered by Tarski in 1966, edited by John Corcoran.

Tarski, A. and S. Givant (1987), *A Formalization of Set Theory without Variables*, Providence (R.I.): American Mathematical Society.

van Benthem, J. (1986), *Essays in Logical Semantics*, Dordrecht: Reidel.

van Benthem, J. (2002), "Logical Constants: the Variable Fortunes of an Elusive Notion", in W. Sieg, R. Sommer and C. Talcott (eds.), *Reflections on the Foundations of Mathematics: Essays in Honor of Solomon Feferman*, Natick (Mass.): AK Peters, 420–440.

Warmbrōd, K. (1999), "Logical Constants", *Mind*, vol. 108, 503–538.

Woods, J. (2014), "Logical Indefinites", *Logique et Analyse*, vol. 227, 277–307.

Woods, J. (2016), "Characterizing Invariance", *Ergo*, vol. 3, 778–807.