however rudimentary, before it becomes the subject matter of science. And this metaphysics goes back to the natural sciences as enumerated above.

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STRICT IMPLICATION—AN EMENDATION

MR. E. L. POST, of the department of mathematics in Columbia University, calls my attention to an error in the development of the system of "Strict Implication," as presented in Chapter V of *A Survey of Symbolic Logic.* The postulate 1.8,

$$(p \prec q) = (\sim q \prec \sim p),$$

is equivalent to the pair,

$$2.2 \quad (p \prec q) \prec (\sim q \prec \sim p)$$

$$2.21 \quad (\sim q \prec \sim p) \prec (p \prec q).$$

Of these, 2.21, "If 'q is impossible' implies 'p is impossible,' then p implies q," is false. It is consistent with the other principles assumed, but is incompatible with the intended meaning of the primitive idea "impossibility," and with the distinction of this from the idea of simple falsity.

Mr. Post's example which demonstrates the falsity of 2.21 is not here reproduced, since it involves the use of a diagram and would require considerable explanation. Suffice it to say that it is entirely convincing. His proof that 2.21 leads to the consequence

$$\sim p = -p$$

is as follows:

$$2.21\colon \quad (\sim q \prec \sim p) \prec (p \prec q) \qquad (1)$$

$$1.02\colon \quad (p \prec q) = \sim(p - q) \qquad (2)$$

$$1.02 \quad \{\sim q/p;\ \sim p/q\}\colon \sim q \prec \sim p = \sim(\sim q - \sim p) \qquad (3)$$

$$(3), (2)\colon (1) = \quad \sim(\sim q - \sim p) \prec \sim(p - q) \qquad (4)$$

$$(1) \quad \{(\sim q - \sim p)/q;\ (p - q)/p\}\colon$$

$$[\sim(\sim q - \sim p) \prec \sim(p - q)] \prec [(p - q) \prec (\sim q - \sim p)] \qquad (5)$$

$$(5)\colon \quad (4) \prec p - q \prec \sim q - \sim p \qquad (6)$$

$$(6) \quad \{-p/q\}\colon p - (-p) \prec (\sim -p)(- \sim p) \qquad (7)$$

$$2.51\colon \quad (7) = p\,p \prec (\sim -p)(- \sim p) \qquad (8)$$

$$2.81\colon \quad (8) = p \prec (\sim -p)(- \sim p) \qquad (9)$$

2.1 $\{\sim -p/p; -\sim p/q\}: (\sim -p)(-\sim p) \prec \sim -p$ (10)

1.6 $\{(\sim -p)(-\sim p)/q; \sim -p/r\}: (9) \times (10) \prec p \prec \sim -p$ (11)

(11) $\{-p/p\}: -p \prec \sim -(-p)$ (12)

2.51: $(12) = -p \prec \sim p$ (13)

1.7: $\sim p \prec -p$ (14)

1.06: $(13) \times (14) = (\sim p = -p)$Q.E.D.

Since the distinction of "impossibility" from simple falsity is essential to that of "strict" from "material" relations, the presence of this consequence of 2.21 would be to reduce the system to a redundant form of "Material Implication."

To correct this error, postulate 1.8 must be replaced by the principle given as theorem 2.2,

$$(p \prec q) \prec (\sim q \prec \sim p),$$

and theorems 2.7, 2.712, 2.72, 2.731, 2.75, 2.76, and 2.77—all of which are alternative forms of 2.21 or 1.8—must be deleted. The proof of the remaining theorems, with the further exceptions to be mentioned immediately, will not be affected; and the important results and general character of the system will still be as presented in the book.

The transformation set forth in Section III, which proves that Material Implication is a subsystem in Strict Implication, can not be carried out in all details in the manner proposed, since theorems 4.3—4.37 of that section involve 2.21 and are invalid. But this transformation can be otherwise effected, as is demonstrated by the fact that all the symbolic postulates for Material Implication given in *Principia Mathematica* can still be deduced. In the proof of these postulates, as given in Section III, the only use of 2.21 or its consequences is in 4.54 and 4.55, which are lemmas to 4.56, and in 4.57. But 4.56 and 4.57 can be otherwise proved as follows:

Lemma 1. $p \prec (q \subset p\,q)$

Proof: 2.4: $p\,q \prec p\,q$ (1)

4.52 $(1) = p \prec (q \subset p\,q)$

Lemma 2. $(p \prec q) \subset (p\,r \subset q\,r)$

Proof: Lemma 1: $q \prec (r \subset q\,r)$ (1)

1.6: $\{(p \prec q)\ [q \prec (r \subset q\,r)]\} \prec [p \prec (r \subset q\,r)]$ (2)

4.15: $[p \prec (r \subset q\,r)] \prec [p \subset (r \subset q\,r)]$ (3)

1.6: $(2) \times (3) \prec \{(p \prec q)\ [q \prec (r \subset q\,r)]\} \prec [p \subset (r \subset q\,r)]$ (4)

4.52: $(4) = (1) \prec (p \prec q) \subset [p \subset (r \subset q\,r)]$ (5)

4.51: $(5) = (p \prec q) \subset (p\,r \subset q\,r)$

Theorem 4.56. $(p \subset q) \subset (p\,r \subset q\,r)$

Proof: Lemma 2 $\{(p \subset q)p/q\}$:

$$\{[(p \subset q)p] \prec q\} \subset \{[(p \subset q)p]r \subset q\,r\} \quad (1)$$

2.91: $(1) = \{[(p \subset q)p] \prec q\} \subset \{[(p \subset q)\ (p\,r)] \subset q\,r\}$ (2)

4.53: $(2) \times 4.53 \prec [(p \subset q)\ (p\,r)] \subset q\,r$ (3)

4.51: $(3) = (p \subset q) \subset (p\,r \subset q\,r)$Q.E.D.

Theorem 4·57. $(p \subset q) = (-q \subset -p)$

Proof: 2·8, 2·51: $-(p-q) = -[-q-(-p)]$ (1)

1·03: $(1) = [(p \subset q) = (-q \subset -p)]$Q.E.D.

For similar reasons, postulate L of the set given for the "Calculus of Ordinary Inference" should be

$$\text{L. } (p\,q \prec r\,s) \prec (p \circ q \prec r \circ s).$$

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REVIEWS AND ABSTRACTS OF LITERATURE

Authority in the Modern State. HAROLD J. LASKI. New Haven: Yale University Press. 1919. Pp. 398.

Usually, we do not understand the institutions we take for granted, and unwittingly we obey Burke's admonition and reverence them. Such has been our attitude toward the state. Of late, when our own political philosophers discussed it, they did so nearly always to justify its existing form of organization. When our political scientists dealt with it, they seldom did more than describe and classify its organs of government.

Mr. Laski breaks with this tradition. His view of the state is heretical, although he hides his non-conformity behind an awe-inspiring mass of pointed references and excellent foot-notes. He inquires into the problem of state authority and the nature of obedience. To Mr. Laski the state is the people organized politically. He would say with William Graham Sumner "the state is all of us," but would add, "yet, not all of each of us." There are innumerable human interests which lie outside the purview of the state, which, after all, is no more than one of the innumerable group units of which society is composed. While the state and government are not identical, it is through government that the state functions, and thus, any real-