NECESIDAD

en S2, S3, S4 y S5

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Si $\vdash_{S2} \alpha$, entonces $\vdash_{S3} \alpha$.

Si $\vdash_{S4} \alpha$, entonces $\vdash_{S5} \alpha$.

Paradojas de la implicación material

$\nvdash_{S3} Q \prec (P \prec Q)$

$\nvdash_{S3} \neg P \prec (P \prec Q)$

$\nvdash_{S3} ((P \wedge Q) \prec R) \prec ((P \prec R) \vee (Q \prec R))$

$\nvdash_{S3} \neg(P \prec B) \prec P$

Tampoco valen en S5.

S2

$\vdash_{S2} \Box\alpha \prec \alpha$

$\vdash_{S2} \alpha \prec \Diamond\alpha$

S3

$\vdash_{S3} \Box\alpha \prec \alpha$

$\vdash_{S3} \alpha \prec \Diamond\alpha$

$\vdash_{S3} (\alpha \prec \beta) \prec (\neg\Diamond\beta \prec \neg\Diamond\alpha)$

S3

$\vdash_{S3} \alpha \prec \alpha$, pero

$\nvdash_{S3} \Box(\alpha \prec \alpha)$

¿No es necesario que algo se implique a sí mismo?

S3

En S3 podemos mostrar que nuestras reglas lógicas son teoremas, pero no podemos mostrar que nuestras reglas lógicas son necesarias. ¿Qué tipo de necesidad es ésta? ¿Es la que queremos modelar?

S4

$\vdash_{S4} \Box\alpha = \Box\Box\alpha$

$\vdash_{S4} \Diamond\alpha = \Diamond\Diamond\alpha$

S4

$$\vdash_K \Box(\alpha \strictif \alpha)$$

S5

$\vdash_{S5} \Box\alpha = \Diamond\Box\alpha$

$\vdash_{S5} \Diamond\alpha = \Box\Diamond\alpha$

$\vdash_{S5} \Box\alpha = \Box\Box\alpha$

$\vdash_{S5} \Diamond\alpha = \Diamond\Diamond\alpha$

S5

$\vdash_{S5} \Box\alpha = \Diamond\Box\alpha$

$\vdash_{S5} \Diamond\alpha = \Box\Diamond\alpha$

$\vdash_{S5} \Box\alpha = \Box\Box\alpha$

$\vdash_{S5} \Diamond\alpha = \Diamond\Diamond\alpha$

S5

$\Box\Diamond\Box\Box\Diamond\Box\alpha =$ ¿?

S5

$$\Box\Diamond\Box\Box\Diamond\Box\alpha = \text{¿?}$$

$\vdash_{S5} \Box\alpha = \Diamond\Box\alpha$

$\vdash_{S5} \Diamond\alpha = \Box\Diamond\alpha$

$\vdash_{S5} \Box\alpha = \Box\Box\alpha$

$\vdash_{S5} \Diamond\alpha = \Diamond\Diamond\alpha$

S5

$$\Box\Diamond\Box\Box\Diamond\Box\alpha = ¿?$$

$\vdash_{S5} \Box\alpha = \Diamond\Box\alpha$

● $\vdash_{S5} \Diamond\alpha = \Box\Diamond\alpha$

$\vdash_{S5} \Box\alpha = \Box\Box\alpha$

$\vdash_{S5} \Diamond\alpha = \Diamond\Diamond\alpha$

S5

$$\Box\Diamond\Box\Box\Diamond\Box\alpha = ¿?$$

- $\vdash_{S5} \Box\alpha = \Diamond\Box\alpha$
- $\vdash_{S5} \Diamond\alpha = \Box\Diamond\alpha$
- $\vdash_{S5} \Box\alpha = \Box\Box\alpha$
- $\vdash_{S5} \Diamond\alpha = \Diamond\Diamond\alpha$

S5

$$\Box\Diamond\Box\Box\Diamond\Box\alpha = ¿?$$

$\vdash_{S5} \Box\alpha = \Diamond\Box\alpha$

$\vdash_{S5} \Diamond\alpha = \Box\Diamond\alpha$

● $\vdash_{S5} \Box\alpha = \Box\Box\alpha$

$\vdash_{S5} \Diamond\alpha = \Diamond\Diamond\alpha$

S5

$$\Box\Diamond\Box\Box\Diamond\Box\alpha = ¿?$$

$\vdash_{S5} \Box\alpha = \Diamond\Box\alpha$

● $\vdash_{S5} \Diamond\alpha = \Box\Diamond\alpha$

$\vdash_{S5} \Box\alpha = \Box\Box\alpha$

$\vdash_{S5} \Diamond\alpha = \Diamond\Diamond\alpha$

S5

$$\Box\Diamond\Box\Box\Diamond\Box\alpha = ¿?$$

- $\vdash_{S5} \Box\alpha = \Diamond\Box\alpha$

  $\vdash_{S5} \Diamond\alpha = \Box\Diamond\alpha$

  $\vdash_{S5} \Box\alpha = \Box\Box\alpha$

  $\vdash_{S5} \Diamond\alpha = \Diamond\Diamond\alpha$

S5

$$\Box\Diamond\Box\Box\Diamond\Box\alpha = ¿?$$

$\vdash_{S5} \Box\alpha = \Diamond\Box\alpha$

$\vdash_{S5} \Diamond\alpha = \Box\Diamond\alpha$

$\vdash_{S5} \Box\alpha = \Box\Box\alpha$

$\vdash_{S5} \Diamond\alpha = \Diamond\Diamond\alpha$

S5

$$\Box\Diamond\Box\Box\Diamond\Box\alpha = \Box\alpha$$

S5

$$\vdash_{S5} (\Diamond\alpha \prec \beta) = (\alpha \prec \Box\beta)$$

$$\nvdash_{S4} (\Diamond\alpha \prec \beta) = (\alpha \prec \Box\beta)$$

¿Implicación en términos de necesidad?

$$\vdash_L \Box\beta \prec (\alpha \prec \beta)$$

$$\vdash_L \neg\Diamond\alpha \prec (\alpha \prec \beta)$$

Si $\vdash_L \alpha$, entonces $\vdash_{S2} \alpha$ y $\vdash_{S4} \alpha$.

¿Implicación en términos de necesidad?

$\vdash_L \Box\beta \prec (\alpha \prec \beta)$

$\vdash_L \neg\Diamond\alpha \prec (\alpha \prec \beta)$

$\vdash_L \Box(\beta \vee \neg\beta)$

$\vdash_L \neg\Diamond(\alpha \wedge \neg\alpha)$

Si $\vdash_L \alpha$, entonces $\vdash_{S2} \alpha$ y $\vdash_{S4} \alpha$.

¿Implicación en términos de necesidad?

$\vdash_L \Box\beta \prec (\alpha \prec \beta)$

$\vdash_L \neg\Diamond\alpha \prec (\alpha \prec \beta)$

$\vdash_L \Box(\beta \vee \neg\beta)$

$\vdash_L \neg\Diamond(\alpha \wedge \neg\alpha)$

$\vdash_L \alpha \prec (\beta \vee \neg\beta)$

$\vdash_L (\alpha \wedge \neg\alpha) \prec \beta$

Si $\vdash_L \alpha$, entonces $\vdash_{S2} \alpha$ y $\vdash_{S4} \alpha$.