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Deviating from the canonical: induced noncommutativity

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Abstract. The motion of a quantum particle, constrained to move along a space curve via a confining potential, is governed by a hamiltonian that depends on the geometry of the curve. Adiabatic changes in the shape of the curve are shown to give rise to geometric phases. The effect of the latter on the quantization of the motion of the curve itself can be naturally described by a deformation of the canonical commutation relations, providing an example of a noncommutative effective quantum field theory.

1. Introduction

At the heart of quantum field theory lie the canonical commutation relations among the dynamical variables of the theory, which, in particular, imply discrete symmetries that experiments have amply verified. For the last two decades or so, several attempts have been made to develop a noncommutative version of the theory, *i.e.*, one where the above commutation relations are somehow deformed. To our knowledge, none of these attempts derives the particular deformation scheme employed from some basic principle, be it physical in nature or mathematical, a state of affairs that leaves the questions of mathematical self-consistency and physical relevance wide open. In the present short communication we study a physical system consisting of a “slow” and a “fast” part in the adiabatic regime, and find that the effective field theory describing the slow part may be cast in noncommutative form, in the sense that a natural set of dynamical variables emerges the commutation relations of which deviate from the canonical. The system consists of a quantum particle constrained to move along a closed space curve by a confining potential with a sharp minimum centered on the curve. For a particular classical configuration of the curve, the 1D effective hamiltonian for the particle’s motion along the curve is found, and its wavefunction is determined. Then the focus shifts on the dynamics of the curve itself, and the effect the particle’s presence has on it. It is found that geometrical phases that accumulate as the curve is deformed, influence its own effective dynamics in that both positions and momenta end up covariantized, satisfying deformed commutation relations. What might be considered an appealing feature of the theory is that the exact form of noncommutativity is unambiguously determined by standard quantum mechanics, the deformed commutators being given by Berry’s curvature.

Regarding the structure of this note, section 2 reviews the basics of constrained quantum mechanics, following [1, 2], in the example of a particle bound by a 2D harmonic oscillator potential in the plane normal to a closed space curve. It is explained furthermore, how cyclic deformations of the curve give rise to geometric phases, as shown in [3]. Section 3 illustrates the mechanism by which the effective

theory for the slow part of a “fast-slow” composite system becomes noncommutative, drawing on an example discussed in [4]. The dynamics of the curve itself is discussed in section 4, while the final synthesis of all the ingredients is outlined in section 5.

2. Particles constrained on a curve and geometric phases

Consider a quantum particle that is forced to live on a closed space curve via a confining potential V , that has a sharp minimum on the curve. For concreteness’ shake, take V to be a 2D harmonic oscillator potential, in the plane normal to the curve. Calling s a parameter along the curve, and a , b , coordinates in the normal plane, along the normal \mathbf{n} and binormal \mathbf{b} of the curve, respectively, we take $V(s, a, b) = (a^2 + b^2)/\eta^4$, where η is a small number, $\eta \ll 1$, that controls the penetration depth of the particle in the normal plane. Switching to rescaled normal coordinates $\alpha = a/\eta$, $\beta = b/\eta$, so that the particle’s wavefunction takes significant values where α , β are of order 1, one obtains for the 3D hamiltonian H of the particle

$$H = -\frac{1}{2|G|^{1/2}}\partial_A G^{AB}|G|^{1/2}\partial_B + \frac{(\alpha^2 + \beta^2)}{2\eta^2}, \quad (1)$$

where G_{AB} is the euclidean metric in the adapted coordinates (s, α, β) . The normalization condition for the 3D wavefunction Φ of the particle is $\int ds d\alpha d\beta G^{1/2}|\Phi|^2 = 1$, where $G = \det(G_{AB})$. In order to derive an effective 1D hamiltonian for the motion of the particle along the curve, we rescale the wavefunction, $\Phi \rightarrow \Psi \equiv e^{-h}\Phi$, with $e^{-h} = (G/g)^{1/4}$ and $g = \rho^2$ the metric of the curve, so that the normalization condition becomes $\int ds d\alpha d\beta g^{1/2}|\Psi|^2 = 1$, and $\int d\alpha d\beta |\Psi|^2$ can be interpreted as the effective 1D probability density for the particle at the position s along the curve, using the conventional measure $g^{1/2}ds$. At the same time, we have to transform the hamiltonian,

$$H \rightarrow \tilde{H} = e^{-h} H e^h, \quad \text{where} \quad \tilde{H} = \frac{1}{\eta^2} \tilde{H}_{-2} + \tilde{H}_0 + O(\eta), \quad (2)$$

and

$$\tilde{H}_{-2} = -\frac{1}{2}(\partial_\alpha^2 + \partial_\beta^2) + \frac{1}{2}(\alpha^2 + \beta^2), \quad \tilde{H}_0 = -\frac{1}{2}(\rho^{-1}\partial_s - \tau L)^2 - \frac{1}{8}\kappa^2. \quad (3)$$

The operator L above is the (antihermitean) generator of rotations in the normal plane, $L = \alpha\partial_\beta - \beta\partial_\alpha$. Factorizing Ψ as $\Psi(s, \alpha, \beta) = \chi(\alpha, \beta)\psi(s)$, and assuming the normal wavefunction χ to satisfy the eigenfunction equations $\tilde{H}_{-2}\chi_{\sigma n} = (n+1)\chi_{\sigma n}$, and $L\chi_{\sigma n} = i\sigma\chi_{\sigma n}$, converts the 3D relation $\tilde{H}\Psi = E\Psi$ to the following equation for ψ ,

$$\begin{aligned} H_0\psi_{\sigma m} &\equiv -\frac{1}{2}(\rho^{-1}\partial_s - i\sigma\tau)^2\psi_{\sigma m} - \frac{1}{8}\kappa^2\psi_{\sigma m} \\ &= -\frac{1}{2\rho^2}\psi''_{\sigma m} + \left(i\sigma\rho^{-1}\tau + \frac{\rho'}{2\rho^3}\right)\psi'_{\sigma m} + \frac{1}{2}\left(i\sigma\rho^{-1}\tau' + \sigma^2\tau^2 - \frac{1}{4}\kappa^2\right)\psi_{\sigma m} = E_{\sigma m}\psi_{\sigma m}, \end{aligned} \quad (4)$$

where κ and τ are the curvature and torsion of the curve, respectively. Notice that the index m in the above equation is discrete, owing to the periodic boundary conditions appropriate for a closed curve. The \tilde{H} eigensystem is then given by

$$\Psi_{\sigma nm} = \chi_{\sigma n}\psi_{\sigma m}, \quad E_{\sigma nm} = \frac{(n+1)}{\eta^2} + E_{\sigma m}. \quad (5)$$

A look at (4) reveals that the shape of the curve, codified in the two functions κ , τ , plays the role of an infinite set of external parameters for H_0 . Cyclic deformations of the curve then, may lead to the appearance of geometric phases in the particle’s wavefunction, and this is indeed what happens,

as was shown in [3]. To calculate the corresponding Berry's curvature, we consider two infinitesimal deformations with parameters $\xi(t)$, $\zeta(t)$, respectively, and calculate the perturbed hamiltonian \mathcal{H} ,

$$\mathcal{H} = H_0 + \xi H^\xi + \zeta H^\zeta, \quad \text{where} \quad H^\xi = i\sigma\tau_\xi\partial_s + \frac{i}{2}\sigma\tau'_\xi + \sigma^2\tau\tau_\xi - \frac{1}{4}\kappa\kappa_\xi. \quad (6)$$

κ_ξ , τ_ξ above are the derivatives of κ , τ , w.r.t. ξ , given by

$$\kappa_\xi = \mathbf{v}''^n \quad (7)$$

$$\tau_\xi = \kappa^{-1}\mathbf{v}'''^b - \kappa^{-1}\tau\mathbf{v}''^n - \kappa^{-2}\kappa'\mathbf{v}''^b + \kappa\mathbf{v}'^b + \tau\mathbf{v}'^t, \quad (8)$$

where $\dot{\xi}(0)\mathbf{v}$ is the deformation velocity field along the curve, at $t = 0$ — similar expressions hold for H^ζ . Next, one computes the eigenfunction $\psi(s; \xi, \zeta)$ of \mathcal{H} and, from it, the rescaled wavefunction $\Psi(s, \alpha, \beta; \xi, \zeta)$ and, finally, Φ . Notice that although Φ is eigenfunction of the cartesian coordinate hamiltonian (1), it is nevertheless expressed in the adapted coordinates. To properly apply Berry's formula for the curvature,

$$K_{\xi\zeta}^{(n)} \equiv -2 \text{Im}(\partial_\xi \langle n, \xi | | \partial_\zeta | n, \xi \rangle), \quad (9)$$

one has to express $\Phi(s, \alpha, \beta; \xi, \zeta) = \langle s, \alpha, \beta | n; \xi, \zeta \rangle$ in the cartesian coordinates, then take the partials w.r.t. the parameters, and reexpress the result in the adapted coordinates, this latter step facilitating the evaluation of the relevant integrals. The same result is obtained if one uses total, rather than partial, derivatives in (9),

$$K_{\xi\zeta}^{(n)} = i \int ds d^2y \sqrt{G} \left\{ \left[(\partial_\xi - \mathbf{V}^\xi) \Phi_n \right]^* \left[(\partial_\zeta - \mathbf{V}^\zeta) \Phi_n \right] - \text{c.c.} \right\}, \quad (10)$$

where $\dot{\xi}(0)\mathbf{V}^\xi$ is the initial velocity field, under the ξ -deformation, of points with fixed adapted coordinates, observed in the ambient \mathbb{R}^3 , as they are dragged along by the moving curve (similarly for \mathbf{V}^ζ), and we have reverted to pre-Dirac notation to show explicitly the integration measure used.

Particularly interesting, for our purposes, is the hamiltonian in the adapted coordinate frame, namely, the generator of time evolution for the adapted frame wavefunction. This can be obtained by transforming the cartesian coordinate time-dependent Schroedinger equation to the adapted frame. Under the change of coordinates $(t, x^1, x^2, x^3) \rightarrow (t', y^1, y^2, y^3)$, with $t' = t$ and $y^i = y^i(x; \xi(t))$, the time derivative in the time dependent Schroedinger equation transforms as

$$\partial_t = \partial_{t'} + \dot{\xi}(\partial y^i / \partial \xi) \partial_{y^i} \equiv \partial_{t'} - \dot{\xi} \mathbf{V}, \quad (11)$$

where the assumption was made that the time dependence of the coordinate transformation is entirely through $\xi(t)$. The term involving $\dot{\xi}$ is to be absorbed in the adapted frame hamiltonian \mathcal{H} ,

$$\mathcal{H} = H + i\dot{\xi} \mathbf{V}. \quad (12)$$

When, subsequently, the wavefunction is rescaled, and all operators undergo a similarity transformation, as in (2), the time derivative of the rescaling factor also contributes to the hamiltonian,

$$\tilde{\mathcal{H}} = \tilde{H} + i\dot{\xi}(\tilde{\mathbf{V}} - \partial_\xi h) = \tilde{H} + i\dot{\xi}(\mathbf{V} - d_\xi h) \equiv \tilde{H} + \dot{\xi} \mathcal{H}^\xi, \quad (13)$$

where $d_\xi h \equiv \partial_\xi h - \mathbf{V}^i \partial_i h$, \tilde{H} is as in (2), and the last equation defines \mathcal{H}^ξ . In the case that two parameters, ξ and ζ , are present, the adapted frame hamiltonian takes the form

$$\tilde{\mathcal{H}} = \tilde{H} + \xi \tilde{H}^\xi + \zeta \tilde{H}^\zeta + i\dot{\xi}(\mathbf{V} - d_\xi h) + i\dot{\zeta}(\mathbf{W} - d_\zeta h), \quad (14)$$

where \mathbf{W} is the 3D vector field corresponding to the ζ -deformation. Notice that $\tilde{\mathcal{H}}$ depends on both the parameters *and* their time derivatives.

There is considerably more that can be said about how the two hamiltonians in (6), (14), predict the same geometric phase, and this despite the fact that the curvatures computed from each differ — we defer a detailed treatment of these matters to a forthcoming publication, currently in progress [5].

3. A toy example of induced noncommutativity

The core idea behind what we call induced noncommutativity is best appreciated in a simplified context, involving a spin 1/2 coupled to the position of an otherwise free particle, a system that has been studied before (see [4] and references therein). The relevant hamiltonian is

$$H_{\text{tot}} = \frac{P^2 + X^2}{2} + \lambda \mathbf{X} \cdot \boldsymbol{\sigma}. \quad (15)$$

Assume the initial state of the spin, at $t = 0$, to be $|\mathbf{X}_0+\rangle$ and that, furthermore, that its evolution can be approximated adiabatically, namely, the motion of the particle is slow enough for the spin to “lock” onto the instantaneous eigenstate $|\mathbf{X}_t+\rangle = \cos(\Theta/2)e^{-i\Phi/2}|+\rangle + \sin(\Theta/2)e^{i\Phi/2}|-\rangle$, where $|\pm\rangle$ are the σ_z eigenstates, and (Θ, Φ) are the particle’s standard spherical angular coordinates. Up to this point, we have treated the particle as classical, with a well defined position \mathbf{X} , which shows up as external parameter in the spin’s wavefunction. Now the operator nature of \mathbf{X} is invoked, and, in particular, its noncommutativity with the momentum \mathbf{P} is taken into account. We calculate the effective hamiltonian for the particle by taking the expectation value of the full hamiltonian in the spin’s assumed quantum state, thus obtaining,

$$H_{\text{eff}} = \langle \mathbf{X}+ | H_{\text{tot}} | \mathbf{X}+ \rangle = \frac{1}{2}(\mathbf{P} - \mathbf{A}(\mathbf{X}))^2 + \Phi(\mathbf{X}) + E^+(\mathbf{X}), \quad (16)$$

where $A_i(\mathbf{X}) = i\langle \mathbf{X}+ | \frac{\partial}{\partial X_i} | \mathbf{X}+ \rangle$ is Berry’s connection and

$$\Phi(\mathbf{X}) = \frac{1}{4}(X^2 + Y^2 + Z^2)^{-1} + \frac{1}{2}(X^2 + Y^2 + Z^2), \quad E^+(\mathbf{X}) = \lambda(X^2 + Y^2 + Z^2)^{1/2}. \quad (17)$$

What most interests us here is the natural appearance of the “covariantized” momentum $\mathcal{P} \equiv \mathbf{P} - \mathbf{A}(\mathbf{X})$, the components of which fail to commute,

$$[\mathcal{P}_i, \mathcal{P}_j] = \frac{\partial A_j}{\partial X_i} - \frac{\partial A_i}{\partial X_j} = -iK_{ij}, \quad (18)$$

where in the second equation, Berry’s curvature has made a suggestive appearance. Starting from this well known example, we propose to dualize the situation, in that we now consider a spin 1/2 coupled to the *momentum* of an otherwise free particle,

$$\tilde{H}_{\text{tot}} = \frac{P^2 + X^2}{2} + \lambda \mathbf{P} \cdot \boldsymbol{\sigma}. \quad (19)$$

Mirroring the above manipulations, the resulting effective hamiltonian is now

$$\tilde{H}_{\text{eff}} = \langle \mathbf{P}+ | \tilde{H}_{\text{tot}} | \mathbf{P}+ \rangle = \frac{1}{2}(\mathbf{X} - \tilde{\mathbf{A}}(\mathbf{P}))^2 + \tilde{\Phi}(\mathbf{P}) + \tilde{E}^+(\mathbf{P}), \quad (20)$$

where $\tilde{A}_i(\mathbf{P}) = i\langle \mathbf{P}+ | \frac{\partial}{\partial P_i} | \mathbf{P}+ \rangle$, and $\tilde{\Phi}(\mathbf{P})$, $\tilde{E}^+(\mathbf{P})$ are given by (17), with $\mathbf{X} \rightarrow \mathbf{P}$. As before, our interest is aroused primarily by the appearance of the “covariantized” position operator $\mathcal{X} \equiv \mathbf{X} - \tilde{\mathbf{A}}(\mathbf{P})$, the components of which, similarly to what transpired above, fail to commute,

$$[\mathcal{X}_i, \mathcal{X}_j] = \frac{\partial \tilde{A}_j}{\partial P_i} - \frac{\partial \tilde{A}_i}{\partial P_j} \equiv -i\tilde{K}_{ij}. \quad (21)$$

Clearly, when the spin’s hamiltonian depends on both the position and the momentum of the particle, the effective hamiltonian for the particle will be naturally expressible in terms of covariantized position and momentum operators, with modified, in general, commutation relations, compared to the canonical ones. We return now to the problem of the particle living on a curve, but focus, this time, on the curve.

4. The dynamics of the curve

We assume the curve is actually a wire, made of an elastic material, naturally shaped into a circle. Small deformations are penalized as in Bernoulli's elastica — the corresponding Lagrangian density is taken to be, in schematic form,

$$\mathcal{L} = \frac{1}{2}\rho\dot{\mathbf{X}} \cdot \dot{\mathbf{X}} - \frac{1}{2}\mu(\delta\kappa)^2 - \frac{1}{2}\frac{\mu}{\lambda}(\delta s)^2, \quad (22)$$

where ρ is a uniform linear mass density, and $\delta\kappa$, δs , are local infinitesimal changes in curvature and arclength, resulting from the above mentioned deformations. Rather than imposing *locally arclength preserving* (LAP) deformations, we let sufficiently small values of λ suppress stretching and compression of the wire. We describe the deviation of the wire from the circular shape by an infinitesimal 3D vector field $\epsilon\boldsymbol{\varphi}(s)$, $\epsilon \ll 1$, the Frenet components $\epsilon(\varphi^t, \varphi^n, \varphi^b)$ of which are the fields \mathcal{L} depends on,

$$\epsilon^{-2}\mathcal{L} = \frac{1}{2}\rho \left((\dot{\varphi}^t)^2 + (\dot{\varphi}^n)^2 + (\dot{\varphi}^b)^2 \right) - \frac{1}{2}\mu (\varphi_n + \partial_s^2\varphi^n)^2 - \frac{1}{2}\frac{\mu}{\lambda} (\partial_s\varphi^t - \varphi^n)^2. \quad (23)$$

The fields are now expanded in Fourier modes,

$$\varphi^i(s, t) = \frac{1}{\sqrt{2\pi}} \sum_k e^{iks} \varphi_k^i(t), \quad (24)$$

the equations of motion of which are diagonalized by the normal mode amplitudes

$$\phi_k^1 = \varphi_k^t - ik\varphi_k^n, \quad \phi_k^2 = -ik\varphi_k^t + \varphi_k^n, \quad \phi_k^3 = \varphi_k^b, \quad (25)$$

with corresponding frequencies

$$\omega_{1k}^2 = \frac{\mu k^2 (1 - k^2)^2}{\rho (1 + k^2)}, \quad \omega_{2k}^2 = \frac{\mu}{\rho} \left(\frac{(1 - k^2)^2}{1 + k^2} + \frac{1 + k^2}{\lambda} \right), \quad \omega_{3k}^2 = 0. \quad (26)$$

The free motion of ϕ_k^3 , implied by the vanishing of ω_{3k}^2 , is an artifact of our perturbative treatment — higher order terms, in ϵ , eventually tame deformations along the binormal. The Lagrangian L may now be obtained, as a function of the normal mode amplitudes, by integrating \mathcal{L} along the circle,

$$\begin{aligned} \epsilon^{-2}L(\phi_k, \dot{\phi}_k) &= \frac{1}{2} \sum_k \frac{\rho}{1 + k^2} (\dot{\phi}_k^{1*} \dot{\phi}_k^1 + \dot{\phi}_k^{2*} \dot{\phi}_k^2) + \rho \dot{\phi}_k^{3*} \dot{\phi}_k^3 \\ &\quad - \frac{\mu k^2 (1 - k^2)^2}{(1 + k^2)^2} \phi_k^{1*} \phi_k^1 - \mu \left(\frac{(1 - k^2)^2 (1 - 2k^2)}{(1 + k^2)^2} + \frac{1}{\lambda} \right) \phi_k^{2*} \phi_k^2 + \mathcal{O}(\lambda). \end{aligned} \quad (27)$$

Finally, we arrive at the hamiltonian of the curve, expressed in terms of the ϕ_k^i and their conjugate momenta,

$$\begin{aligned} \epsilon^{-2}H(\phi_k, \boldsymbol{\pi}_k) &= \frac{1}{2} \sum_k \frac{1 + k^2}{\rho} \pi_k^{1*} \pi_k^1 + \frac{1 + k^2}{\rho} \pi_k^{2*} \pi_k^2 + \frac{1}{\rho} \pi_k^{3*} \pi_k^3 \\ &\quad + \frac{\rho \omega_{1k}^2}{1 + k^2} \phi_k^{1*} \phi_k^1 + \frac{\rho \omega_{2k}^2}{1 + k^2} \phi_k^{2*} \phi_k^2 + \rho \omega_{3k}^2 \phi_k^{3*} \phi_k^3 + \mathcal{O}(\lambda), \end{aligned} \quad (28)$$

where $\pi_k^i \equiv \epsilon^{-2}\partial L/\partial\dot{\phi}_k^i$ are given by

$$\pi_k^1 = \frac{\rho}{1 + k^2} (\dot{\phi}_k^1)^*, \quad \pi_k^2 = \frac{\rho}{1 + k^2} (\dot{\phi}_k^2)^*, \quad \pi_k^3 = \rho (\dot{\phi}_k^3)^*. \quad (29)$$

5. Backreaction and induced noncommutativity

We are finally ready to consider the backreaction of the particle on the curve. As the technical aspects of the discussion quickly degenerate into long formulas, we concentrate in this section on the conceptual side of the problem, leaving a detailed analysis to a lengthier publication, currently in progress. We begin by examining the cartesian coordinate description, in which the particle's hamiltonian is given by (6) while the curve's by (28). It is crucial, for our discussion, to realize that the parameter ξ in (6) can be made to coincide with one of the Fourier modes φ_k^i of (24) if the corresponding deformation is taken to be an exponential. Needless to say, a precise statement of the above idea would necessitate working with real deformations, as those assumed to accompany ξ , but this can of course be easily arranged by working with a Fourier expansion in sines and cosines, instead of complex exponentials — in the loose spirit of this section advertised above, we bypass details of this nature. Then, the particle hamiltonian depends on the curve's Fourier amplitudes φ_k^i , or, which is the same, the normal mode amplitudes ϕ_k^i , and we are essentially in the same situation as with the spin-coupled-to-position example of section 3. When the effective hamiltonian for the curve is computed, by taking the expectation value of the full hamiltonian in the particle's assumed quantum state, the normal mode amplitudes that appear as external parameters in the particle's wavefunction, will be promoted to their full quantum operator status, and will give rise to covariantized conjugate momenta, the commutator of which will involve Berry's curvature, as given in (10). Thus, the effective hamiltonian for the curve, in the presence of the particle, can be naturally expressed in terms of the normal modes ϕ_k^i and covariantized conjugate momenta $\tilde{\pi}_k^i$, which do not commute among themselves. Of course, one may as well choose to work with the original canonical variables instead, so that the induced noncommutativity, although quite natural, is entirely optional. The physical system under study then, provides us with a laboratory where noncommutative field theory can be explored, and its techniques tried and refined, while an alternative commutative treatment is available all along, so that all results can be checked against those obtained by standard techniques.

The situation is even richer, when the analysis is carried out in the frame adapted to the curve. Then the particle's hamiltonian is $\tilde{\mathcal{H}}$, as given in (14), and clearly depends not only on $\xi \sim \varphi_k^i$ but also on $\dot{\xi} \sim \pi_k^i$. Accordingly, noncommuting normal mode amplitudes $\tilde{\phi}_k^i$ will also naturally appear in the curve's effective hamiltonian, and a fully noncommutative field theory will be available to explore. In this case, all canonical commutation relations among the dynamical variables, are, in principle, modified, providing a concrete deformation scheme with guaranteed self consistency, originating, as it is, in standard quantum mechanics.

6. Concluding remarks

We have shown, in this short note, that the presence of a quantum particle, constrained to move along a closed space curve, induces, in the adiabatic limit, magnetic potential-like terms in the curve's effective hamiltonian, which are naturally absorbed in covariantized, and hence noncommuting, in general, dynamical variables. The availability, in parallel, of the commutative treatment, allows constant control over results obtained by novel techniques. The fact that the resulting deformation scheme originates in standard quantum mechanics resolves from the very beginning possible consistency doubts, and provides a paradigm of sensible noncommutativity. Given the modified commutation relations among the covariantized normal mode amplitudes $\tilde{\phi}_k^i$, it is particularly interesting to determine the corresponding modifications in the commutators among the deformation fields $\tilde{\varphi}^i(s) = \sum_k \tilde{\phi}_k^i e^{iks} / \sqrt{2\pi}$ (compare with (24)).

The nature of the results obtained here is such that one can reasonably expect them to be true generically: in the quantum world, physical systems are invariably accompanied by "parasites", like the cloud of virtual particles hovering over, say, a soccer ball in flight. What is to be inferred from the above analysis, is that the effective description of a great number of physical systems might well involve a natural noncommutativity, so that noncommutative field theory, instead of a passing two-decade fad, might be here to stay.

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