My disagreement with Daniell Macbeth

Axel Arturo Barceló Aspeitia
abarcelo@filosoficas.unam.mx

Abstract. Euclidean diagrams are a s*ui-generi*s case among scientific representations, and there is still ample debate as to whether they are symbols, indexes or icons, and of what sort. I hold them to be pictorial icons that reproduce at least some visual features of their objects. This hypothesis has been directly challenged by Sherry (2009) and Panza (2012) among others. I want to focus this paper on defending this thesis against Macbeth's (2009, 2010, 2014) claim that Euclidean Diagrams are fruitful in the discovery, understanding and proof of geometrical facts because of their homomorphism with genuine geometrical objects. This means that visual similarity between geometrical object and diagram, even if it existed, would play no role and would instead be just a byproduct of structural similarity. In response, I will argue that, even if structural similarities of the sort MacBeth identifies are sufficient to account for Euclidean diagrams' inferential role, visual similarities are still important to account for how we interpret diagrams. In particular, I will argue that a pragmatic account off diagrammatic content complements well MacBeth's main epistemological theses without excluding the importance of visual resemblance.

Keywords: Euclidean Diagrams, iconic representation, resemblance.

1. Introduction

Philosophers have long been intrigued by the many devices we have developed for representation and communication and, in particular, by the striking differences between words and pictures. However, pinning down the exact difference between them has proved to be elusive, to say the least. These are some of the many ways philosophers have proposed for distinguishing (at least some sorts of) pictures from words just in the last few decades:

- By their content: Words have conceptual content, pictures have non-conceptual content
- By their persuasive force: Words are apollinean, pictures are dionisian
- By their structure: Words have a recursive syntatcic/semantic structure, pictures are dense
- By what makes them representations: Words belong to languages, pictures are autonomous depictions
- By how they are related to what they represent: Words are artificially related to what they represent, pictures are naturally related to what they represent

- By how we grasp their content: The content of words is interpreted according to linguistic conventions plus contextual information, the content of pictures is seen in them
- By their modality: Pictures are visual, words need not be
- By their phenomenology: Seeing a picture of X feels similar to seeing or being in the presence of X itself; reading a word meaning X, less so.

Thus, it has become uncontroversial to say that words and pictures are not fine enough categories for the study of representation and thought; that that we need, as philosophers and semioticians, to develop new vocabularies and to draw new, finer distinctions. That is why some philosophers have developed and adopted technical distinctions like Grice's distinction between natural and artificial meaning, Peirce's distinction between symbols, icons and signals, etc. The idea is to notice that some pictures may differ from words in some aspects, while other sorts of pictures might differ in other, different respects.

I have adopted Peirce's distinction between symbols, signals and icons to address the fifth (and sixth) of the above questions. Unless I am mistaken in my reading of Peirce, his notion son "symbol" and "index" roughly correspond, on Grice's distinction, to signs that have artificial and natural meaning, respectively. Icons are interesting, therefore, because they hold an interesting middle position between natural signals and artificial symbols. Words are paradigmatic symbols, for they commonly have no natural relation to what they stand for. The word "dog" has no natural relations to dos. Instead, the relevant semantic relation holds artificially through some sort of intentional, stipulation that becomes a socialised convention of use. On the other side, footprints are paradigmatic examples of signals, for they are naturally linked to what they carry information about. The footprint of a wildcat in the dirt is causally related to the wildcat whose presence it signals, and it is because of our knowledge of this causal link that we can infer one from the other. However, we must not read too much into the 'natural' moniker and think that the relation between signal and what it carries information of is always causal, unless we want to exclude structures and other abstract entities by definition.

Now, Peirce originally introduced the notion of an icon on his 1867 paper "On A New List of Categories" to classify representations linked to their objects via "a mere community in some quality" or likeness (p. 56). Paradigmatic examples of icons are realistic pictures, and other depictions. This sort of pictures hold a middle ground between symbols like words and signals like footprints. Like symbols, they represent what they represent by an artificial and intentional act – the act of artificially reproducing the visual appearance of its object –, but like signals they relay on something that is naturally linked to what they depict – the appearance they reproduce. However, icons of other sorts right reproduce other aspects of their object, for example, many scientific models reproduce structural features of their target systems.

Euclidean diagrams are an interesting case of scientific representations, because there is still ample debate as to whether they are symbols, indexes or icons, and of what sort. Brown (2008) famously conceived them as windows into the platonic realm of mathematical objects. I interpret this as taking diagrams to be indexes non-artificially linked to the mathematical facts they give us epistemic access to. Kuvlicki

(2010) has argued that they are governed to syntactic and semantic conventions very much like languages. Giardino (2017), MacBeth (2009) and French (2003) hold that they are structural icons, i.e., they are fruitful in the discovery, understanding and proof of geometrical facts because of their homomorphism with genuine geometrical objects. In contrast, I hold them to be icons that reproduce at least some visual features of their objects. This hypothesis has been directly challenged by Sherry (2009) and Panza (2012) among others. I have addresses some of these challenges elsewhere, and want to focus this paper on defending this thesis against Macbeth's (2009, 2010, 2014) claim that Euclidean Diagrams are fruitful in the discovery, understanding and proof of geometrical facts because of their homomorphism with genuine geometrical objects. This means that visual similarity between geometrical object and diagram, even if it existed, would play no role and would instead be just a byproduct of structural similarity. In response, I will argue that, even if structural similarities of the sort MacBeth identifies are sufficient to account for Euclidean diagrams' inferential role, visual similarities are still important to account for how we interpret diagrams. In particular, I will argue that a pragmatic account off diagrammatic content complements well MacBeth's main epistemological theses without excluding the importance of visual resemblance. According to Macbeth (2009, 2014), geometrical diagrams can be successfully used to prove theorems about geometrical objects they do not resemble and thus cannot be pictures of. I will argue that Macbeth's arguments misconstruct the role resemblance plays in icon and thus presents no challenge to my main thesis.

2. Inference and Representation

Judges use a photo finish to determine who won a race, a driver stops at a corner to ask a passer by for directions, a radiologist examines a patient's x-ray before giving diagnosis, a traveller checks the screen at the airport to get information about her flight, a scientist checks the reading on her nanometer to determine the length of her samples, a mathematician looks at a diagram to gain insight into a new conjecture, etc. What all these cases have in common is that in all of them a person tries to get information about the world not by direct observation but by the use of representations. In every case, the information might be more or less accurate, the method we use more or less reliable, but in all of them the information is mediated by a representation: a photograph, some words, an x-ray, etc. Because of the mediating nature of representations, in every case, the person goes through two different cognitive processes in order to get the information she wants: she determines both the content of the representation, and also whether she ought to trust the representation and, therefore, incorporate the content of the representation to her own system of beliefs about the world or not. The lab scientist must know both how to use her instrument in order to get a reliable reading, but she must also know how to read it to extract this information from it. An error in either process could result in a bad belief, either false or unjustified.

When dealing with episodes like this, the philosopher is interested in determining what epistemic advantages we glean from the use of representations, and why are we justified in doing so. For example, why do we sometimes need to check photographs to determine who crossed the finish line first at a race that occurred just right in front of our very own eyes?, and second, why were we justified in accepting the conclusion we thus reached? In general, when talking about the successful role of representations in knowledge, questions of these two same sorts always arise. They correspond to what elsewhere I have called their "logical" and "ergonomic" dimensions (Barceló 2016). These two sorts of questions require different sorts of answers. For example, answering the informational question regarding the use of photo-finish might require saying something about the causal process behind photography and maybe also something about the location of the cameras in relation to the finish line. In contrast, answering the second, cognitive question might require saying also something about the limits of our perceptual system and thus why we could not see the winner with the naked eye, etc. and thus, why there is an ergonomic advantage in using photographs for this sort of purposes.

Consequently, a proper philosophy of diagrammatical reasoning in geometry must address not only the question of why (and when) are geometrical diagrams reliable means for making inferences about the geometrical realm (Mumma 2010, Krummheuer 2009, Kulpa 2009, Brown 2008, Guiaquinto 2007, Lomas 2002, Norman 2006, Shimojima 1996, etc.), but also why they are useful for doing so (Blackwell 2008; Giardino 2012). We must expect that what makes a geometrical diagram helpful for a given proof be not only its accuracy in representing a geometrical object or state of affairs, but also its cognitive benefits: its tractability, accessibility, clarity, etc. In other words, a good diagram must not only be effective in giving us the information we need, with as little noise as possible, but must do so in an efficient way.

In what follows I will argue that partly, why we use diagrams in geometrical proof is similar to why we use photographs taken at the finish line of races to decide who won, i.e., because diagrams, like photographs, visually resemble what they represent. It is a truism that, in appropriately using a representation to make an inference, it is important to be able to identify what is being represented. There is ample empirical evidence that, all things being equal, it is desirable that representations be developed in such a way that their referents are easy to identify (Paraboni et al. 2007). However, there are as many different ways of making referents easy to identify as there are mechanisms of reference. One of the most common is the establishment of a convention through some kind of "baptism" (Kripke 1980), but there are others. For example, we identify the referentes of audio recordings, realistic drawings and sculptures, etc. because we identify certain important perceptual similarities between them and what they stand for. For instance, I can recognise my mother's voice on the phone because, even if the signal is degraded, there are enough similarities between her actual voice and the sounds emanating from the phone's speakers for me to match one to the other. My main claim here is that geometrical diagrams are like pictures or recordings in this regards, i.e., our interpretation of them is also guided by resemblance.

Pictorial icons have a clear cognitive advantage over other kinds of representations: when determining what something represents, it helps a lot if the representation looks similar to its referent. If we look back at the photo finish example above, we will see that even though there could be other mechanisms that could accurately report the information of who crossed the line first, the photo finish has become a standard mechanism partly because among its practical and cognitive advantages, photographs look like what they represent. What we see when we see a photo finish is pretty similar to what we would have seen if we could have seen the final instant of the race frozen in time in front of us. This makes the information the picture contains about the race easily accessible, and its reliability very vivid. This means that part of why we epistemically use photographs in cases like this is precisely because they look like what they represent. The main thesis I will defend against Macbeth's challenge is that this is is also true about geometrical diagrams: they also look like what they represent and, this is partially why they succeed in representing the geometrical objects they do. When we draw a more-or-less straight line in a diagram, the default assumption is that it represents a straight line, just as if we are asked to represent a triangle, we make our best to draw something that looks like a triangle.

3. Macbeth's Challenge

A challenge to the hypothesis that diagrams are pictorial icons has been raised by Danielle Macbeth on her excellent study of Euclidean diagrams (Macbeth 2009, 2010 and 2014). According to Macbeth, "in Euclidean demostration' a drawn circle in Euclid is not usefully thought of as giving us a picture or instance of the thing that the word "circle" names." (2011, 62). For her, the role of diagrams in mathematical proof largely consists in the de- and reconfiguration of content displayed by geometrical drawings, not in the analysis of a given static picture. Consider, for example, the proof of Euclid I,1.

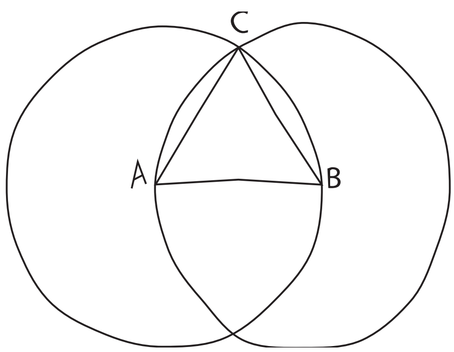


Fig. 1. Euclides I.1

Notice how in order for the proof to go through, it is essential that we are able to regard one and the same drawn line AB now as the radius of a circle and then as one side of a triangle. According to Macbeth, this is incompatible with the thesis that geometrical diagrams are icons, since the content of a icon cannot change in the course of reasoning about what it represents (Macbeth 2009, 252). According to Macbeth, if diagrams were icons, no diagram representing one single object could be used to draw conclusions about other, different objects. A diagram of an isosceles triangle, for example, could be used to draw conclusions about isosceles triangles or about triangles in general, but not about circles or pyramids. However, argues Macbeth, there are clear cases of mathematical proof in Euclid, like the aforementioned Euclides I.1., where this is not what happens. Consequently, for a diagram to play a role in mathematical proof like the one the diagram plays in Euclid I.1., it has to allow for shifting content. According to Macbeth, this is incompatible with the thesis that geometrical diagrams are pictorial icons, since the content of a pictorial icon cannot change in the course of reasoning about what it represents (Macbeth 2009, 252). Thus, we need an account of the content of Euclidean diagrams that allows for shifts in content and for what she calls, following Manders (1996, 2008), the popping up of new geometrical information from the diagram.

> In a Euclidean demonstration, what is at first taken to be, say, a radius of a circle is later in the demonstration seen as a side of a triangle. But how could an icon of one thing become an icon of another? How, for example, could an icon of a radius of a circle turn into an icon of a side of a triangle? (MacBeth 2009, 252)

To account for the shifting content of Euclidean diagrams, Macbeth endorses a pragmatic account where the author's intensions, as manifest in the diagram's accompanying text, play an essential role in determining its content. According to her,

> ...the Euclidean diagram can mean or signify some particular sort of geometrical entity only in virtue of someone's intending that it do so and intending that that intention be recognised. One's intention in making the drawing—an intention that can be seen to be expressed in the setting out (in those cases in which there is one) and throughout the course of the *kataskeue*—is, in that case, indispensable to the diagram's playing the role it is to play in a Euclidean demonstration. (Macbeth 2014, 82)

Furthermore, one's intention can override what the diagram shows, so that if the geometer draws an angle with the intention "merely to draw an angle,... that which he draws... will necessarily be right, or acute, or obtuse; but [what it represents] will be neither right nor acute nor obtuse. It will simply be an angle." (Macbeth 2014, 82)

According to Macbeth, the recognition that intensions play an essential role in determining the content and role of a diagram entails that they cannot be pictorial icons, i.e., that a figure drawn in a Euclidean diagram, even though it may also resemble its object in appearance, it does not represents it in virtue of this resemblance in appearance. (Macbeth 2014, 95) Macbeth recognises that diagrams of circles, for example, look like circles, but argues that it is not because of this that they represent circles, but because of the interplay between pragmatic mechanisms and a structural homomorphism between the parts of the diagram and those of the geometrical objects it represents.

Thus, for Macbeth, an account of diagrammatic representation that takes seriously the importance of intensions is incompatible with the hypothesis that diagrams are pictorial icons (Macbeth 2009, 252-3).

> Drawn figures in Euclid do not just picture various geometrical figures (any more than Arabic numeral picture collections of things); instead they display the contents of the concepts of figures in plane geometry, themselves understood in terms of relations of parts, in a mathematical tractable way. A drawn circle in Euclid is not just a picture or instance of a circle but instead an iconic display of the relation of parts that is constitutitve of something being a circle. (macbeth 2011, 15)

In the following, I aim to show that this is not so, i.e., that Macbeth is wrong in thinking that the content of icons is fixed previously and independently of any pragmatic considerations regarding the intensions of the mathematician. On the contrary, I will show that an adequate account of the interpretation of pictorial icons, in general, ought to incorporate intentional concerns and, therefore, that the hypothesis that diagrams are pictorial icons not only is compatible with Macbeth's recognition that intensions play an essential in determining a diagram's content, but actually predict it.

Macbeth is completely right in stressing the importance of the structural homomorphism between diagrams like the one in Euclid I.1 and their geometrical targets, because it is because of this homomorphism that their use in proof is of epistemological value. However, as I have tried to stress since the beginning of this article. This is just half of the story. We also need to explain why they also satisfy the cognitive constraint, and here is where I think structural accounts like Macbeth fall a little short. By ignoring the more practical constraints imposed on our use of diagrams, they fail to recognise that diagrams are icons, as I will try to show now.

4. A Pragmatic Account of Pictorial Icons

To say that resemblance guides our interpretation of pictorial icons means that resemblance is a necessary, but not sufficient, condition for something to be their referents. In consequence, resemblance underdetermines pictorial iconic representation: how a pictorial icon looks is never sufficient to determine what it represents. I will base my answer to Macbeth's challenge on this basic insight. Even though there is a broad debate regarding exactly what it takes for something to represent something else, there is a growing consensus in the philosophical literature that, at least in the case of of what Grice once called non-natural meaning – and Macbeth recognises that diagrams have non-natural meaning in this very sense –, intention and context are also heavily involved in the interpretation of most linguistic and non-linguistic representations (Schier 1986; Bantinaki 2008; Blumson 2009; Abell 2005, 2009; etc.).

In general, a pictorial icon *p* represents an object or state of affairs *o* iff *p* was made to resemble (or was selected because of its resembling) object *o* in such a way that under normal conditions, the audience of the representational act is able to work out that *p* represents *x* on the basis of what *p* looks like, the rational assumption that it was used with a representational purpose (i.e. that it was used to represent some particular object, or objects, to a certain audience), and other background assumptions (about how the represented objects look like, about the conventions of the media employed, etc.). This is accomplished, most commonly, if the audience realises that it would be very unlikely that the user would have given its representation the appearance it has (resembling object *o*) unless she wanted us to recognise it as representing object *o*.

On this account, a stick figure, for example, can be used to represent a person (to a particular audience in a given context), if it would be rational to expect from such an audience, in such a context, to figure out that, once assuming that the stick figure was used with the intention of representing something and assuming certain background information, both about how people look, and about what resources were available to the user (for example, how much time she had to make such a drawing), that her most likely intention in making it look like a person was to represent a person.

One must be very careful in noticing that to say that a representation looks like or visually resembles its subject does not mean that looking at the representation *is just like* looking at its subject. All it means is that there are some properties that the icon shares with its subject that are also visible in the icon. This means that for the desired resemblance relation to hold it is not necessary that the same property that is visible in the icon is visible also in the subject. When one thinks of everyday icons, i.e., photographs, figurative drawings, etc., the properties of the subject reproduced in the icon are properties that one can also see in the subject. However, not all icons are like that. As a matter of fact, many of the icons used in science are not like that. In many of them, what we see in the icons could not be seen directly on the represented things themselves. Micrographic, telegraphic and stroboscopic pictures, for example, let us see things that are not visible to the bare eye. Consider the aforementioned example of photo finish photography: there is a sense in which what we see is similar to how the last instant of the race would look like, but of course we cannot actually see such instants (Canales 2009).

In consequence, when I claim that a diagram D resembles a geometrical object O all I claim is that there is at least one property P such that (i) both D and O are P and (ii) one can perceive that D is P. Consequently, when I say that D represents O in a proof, all I mean is that D represents O at least partially in virtue of being drawn so as to share this property P in a way that under normal conditions, the readers of the proof can be rationally expected to be able to use this information to work out its content – i.e., that it represents O (most likely, because, in the context, O is the most relevant object to have P).

As I have previously argued, when aiming to represent a geometrical state of affairs in a proof, one must decide both what must be represented and how. To determine what must be represented, one must consider what is given in the initial conditions of the proof, what has already been proved, etc. To determine how should one represent it, one has to evaluate the informational and cognitive advantages and disadvantages of the different means of representation available. In particular, one must decide what information is worth representing in the diagram, and what information is better left in the text. To determine whether a feature of the target geometrical situation is worth reproducing in the diagram, one must weigh both its cognitive and informational costs and benefits. In Euclid's Theorem I.1., for example, we use a roundish closed curve to represent a circle, instead of a perfect circle, because drawing it perfectly round would require too much effort without adding much new relevant information. On the other, if the line was not closed, but open, it would lack a feature of circles key to the validity of the proof (as Macbeth clearly states) and if it was not roundish but polygonal, the resulting diagram would have been too confusing. Thus, we conclude that being a roundish closed curve is a feature of the circle that is worth reproducing in its icon, while perfect roundness is not.

Whatever constraint we do not include in our diagram, still has to be communicated in considered in the proof. This is commonly done textually, but there are other mechanisms we can also use. For example, in current practice, a little quadrangle is usually added on the angles that are to be interpreted as straight. These symbols are not part of the icon itself, but auxiliary symbols that are added on top, just as the letters used to identify the angles.

In the end, what features we decide to include in the icon will depend on the costs and benefits of including or excluding them. When we cannot include in the digram all the information given in the setting of the problem, we have to choose which information to exclude and make sure there are enough indications, either in the accompanying text or symbols, as to what information is missing from the diagram. This is why the resemblance between icons and their referents is rarely total: Most of the times, it is not worth reproducing all the properties of the represented object in the icon; it might even be disadvantageous. Accordingly, most icons have properties (including perceptual ones) that their represented subjects do not have, and vice versa. This is why, for example, two-dimensional pictures can be used to represent three-dimensional objects, black and white images can be used to represent coloured objects and diagrams in Euclidean space can be used to represent figures in Non-Euclidean space. In general, for any property *F*, objects that are not *F*, and do not even look like being *F*, can be successfully used to represent entities that are *F* as long as there are other similarities and contextual clues that allow the interpreter to identify the icon's content. How context helps the interpreter of a diagram fix its referent will be explained in detail in the following section.

5. Context and Interpretation

According to the account of iconic representation and its interpretation I have developed so far, how a picture looks is just part of the information the interpreter exploits in order to determine what is being represented. Consequently, what a picture represents strongly depends on its context of use. As Calderola (2010), Dilworth (2008), Bantinaki (2008), Hyman (2006) and many others have insisted, visual resemblance is a many-to-many relation, i.e. different images may resemble the same object, and the same image can resemble many objects. As such, visual resemblance may restrict the kind of objects a picture can represent, but it cannot always determine what it is being used to represent in every situation of use. Determining the content of a icon is not a matter of determining what it resembles the most. Extra background information is usually necessary, and depending on what background or contextual information is given, the same icon can fix on one referent or another. This is why, in geometry, the same figure can be used to represent different entities or states of affairs in different contexts.

To explain how context helps fix the content of icons in their interpretation, it might prove helpful to say a little bit more about how context is exploited in human communication. For the purposes of this paper, let me adopt the well known account owed to H. Paul Grice (1975), according to whom, whenever we engage in conversations, our communication is guided by a set of assumptions or maxims. These maxims include: (maxim of quality) say only what you believe to be true and of which you have enough adequate evidence; (maxim of quantity) be as informative as necessary, (maxim of relation) contribute only relevant information to the conversation; (maxim of manner) and be clear. These maxims together conform what is known as the cooperative principle: "Make your conversational contribution such as required, at the stage at which it occurs, by the accepted purpose of the talk exchange in which you re engaged" (Grice 1975, 46). Appealing to this maxim has proved to be helpful in explaining how we exploit contextual information to resolve ambiguities, fix extension to predicates, understand sarcasm, etc. Assume now our use of diagrams follows Grice's cooperative principle. In particular, assume Euclid's use of figure 1 to illustrate, among other geometrical facts, that AB and AC are radii of the same circle with center A. As drawn, points B and C stand on a closed curve surrounding point A. This curve resembles a circle, but it also resemble (among other things, and to a lesser degree) other sorts of curves and therefore could be used to represent them. Thus, it is necessary to consider which of these possible interpretations is most likely to be the one intended by the author. Without further information, the most promising hypothesis is that the diagram represents a simple figure very much like itself, i.e., a circle. Furthermore, after reading the accompanying text, we realise that this was the author's representational intention. Thus we infer that points B and C lie on the circumference of a circle entered at A.

Sometimes, however, in order to provide a consistent interpretation of the diagram that takes in consideration both what the text says and how the diagram looks, one must reject not what the diagram shows, but what the text says instead. Consider for example, figure 7 as used in Euclid's reductio proof in I.6 (See Netz 1999, p. 55).

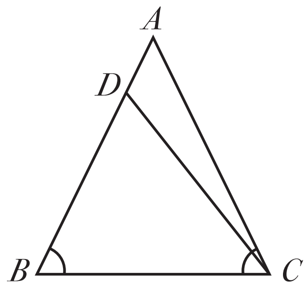


Fig. 2. Euclid I.6

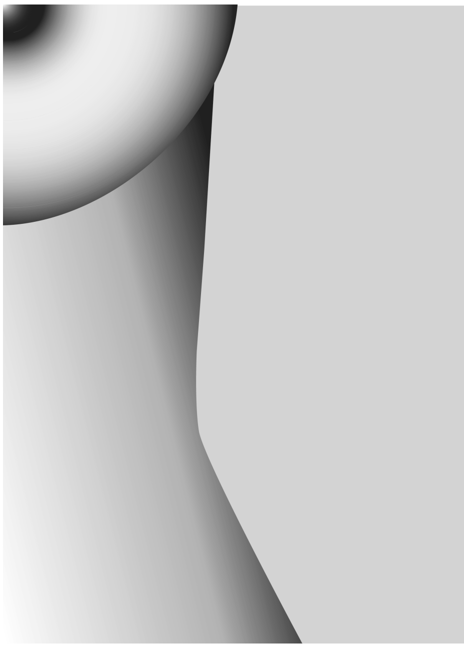
In the diagram we see two triangles sharing one side (BC) and one angle (DBC), as sated in the initial conditions of the proof. We also see that one of the triangles (BCD) is inside the other (ABC) and, consequently, is smaller. Finally, we also see that angles ABC and ACB are more or less equal. The accompanying text confirms that the angles they represent are equal. It also asks us to work under the hypothesis that DB = AC. Thus, we assume that the represented triangles ABC and BCD are of different sizes, have two equal sides (BC = BC and DB = AC) and one equal angle (ABC = DBC). However, we know from previously proved results, that if two triangles have two equal sides and one equal angle, one cannot be larger than the other, which contradicts what we see in the digram. We have reached a contradiction. Since we need to restore consistency in order to determine the diagram's reference, and the contradiction is easily avoided if we reject the hypothesis under consideration, we do that. Once we stop trying to interpreting lines DB and AC as equal in length, we can easily identify the represented figures as two triangles ABC and BCD such that ABC > BCD, ABC = DBC, BC = BC and AB > DB. These, of course, are not impossible triangles, but regular possible triangles. This way, we can make sense of what happens in reductio proofs without having to postulate impossible geometrical objects. In general, in reductio proofs of this sort, the diagram does not represent the hypothesis to be reduced (or the contradiction reached from it), but the positive conclusion we obtain from the reductio. If a reductio proof assumes that not-P to get to a contradiction and thus show that P, we can expect its diagram to represent a situation where P holds, not one where the reduced hypothesis not-P holds, for this is impossible.

For Macbeth, "one cannot *picture* something that is impossible" (2011, 63) and, thus, the diagrams in Euclidean *reductio* proofs cannot be pictorial; however, I hope to have shown that what is depicted in these diagrams is not the impossible state of affairs to be reduced, but the very situation that the reductio aims to prove, which is not impossible at all!

Disregarding the underdetermination of icon by resemblance makes it hard to understand how the same diagram (or different tokens of the same diagram type) could be used to represent different mathematical objects. Euclid's diagram for I.6 is mysterious only under the wrong impression that it is sufficient to look at a diagram to get to its content (Larkin and Simon 1995). Yet, once we recognise the importance of context in determining what is represented, we realise that there is nothing mysterious in it. Without a proper understanding of the role of contextual information in the interpretation of icons, one might not see how the diagram can change its content from one quadrangular to another. So, he takes the radical anti-realist and anti-representationalist alternative of claiming that mathematical diagrams (and mathematical formulae) do not represent mathematical objects at all. However, once we understand that how a diagram looks underdetermines what it represents, no such radical move is required. Drawing only one parallelogram, all three parallelograms can be represented. Thus, Macbeth's challenge poses no real threat to the thesis that geometrical diagrams are icons.

6. Creative ambiguity and seeing-as

Let me now take some time to develop the thesis that the kind of shift in content that is required for proofs like Euclid I.1. can be accounted within a framework like mine, i.e., that recognising that the use of diagrams in Euclid requires actually seeing the same diagram (or parts of a diagram) as different things in different moments, is compatible with my claim that diagram are icons, for icons can also be seen in different ways, i.e., sometimes to adequately interpret an image, it is necessary to recognise a shift in what it represents. In other words, what Grossholz has called "creative ambiguity" is a feature common to icons, thus it is far from being incompatible with it. Even taking into consideration the conventions and techniques at play, a single image can equally resemble more than one object and ambiguity may result, and this ambiguity can be exploited in the conveying of a message. Consider the following example: In 2004, the Light of Life foundation ran a series of ads with the purpose of raising awareness of the growing number of cases of neck cancer among women. The ad featured a cropped photograph resembling a woman's naked torso, showing part of her waist, one of her breasts and just the edge of her nipple. However, the image was actually a cropped photograph of a woman's face, showing just her lower lip, chin and long neck. Even if we take in consideration the usual conventions associated with color photographs, without further input from its context, it is impossible to determine whether it represents a fragment of woman's naked torso (part of her waist, one of her breasts and just the edge of her nipple), or a fragment of a woman's face (a tip of her lower lip, a



quarter of her chin and half her long neck). It resembles both things equally. The image also contained the legend "The fastest growing cancer among women is not what you think". The photograph, by Frank W. Ockenfels, was purposely ambiguous between both interpretations – torso and neck – and it was this ambiguity that made it a perfect fit for harnessing the message that the foundation wanted to communicate: that the fastest growing cancer among women is not what most people think, i.e., breast cancer, but neck cancer. In order to successfully interpret the ad, it is necessary to interpret the photograph as representing once a torso and then a neck. This might require seeing, for example the same crimson part both as a nipple and then as a lip, but this does not make the picture no longer a icon.

Thus, the possibility of creative ambiguity is not only consistent, but a consequence of the fact that isolated from their context, pictures do not represent anything, but only when placed in an adequate context. In other words, substantial input from the context is necessary to determine what the image represents. If the context changes, what the picture represents might change as well. If the context is dynamic, as it is in geometrical proofs

Given the importance of contextual information in determining the content of icons, that the same icon can change content from one context to the next must not be surprising at all. Thus, when we have a dynamic context like in the example above, it is not surprising that the same icon switches from representing a torso to representing a neck, from representing a nipple to representing a lip. Similarly, when the proof so requires, we might be able to interpret the same line as representing a side of a triangle one time, and as representing a radius of a circle.

This phenomenon has been tried to be explained in terms of seeing as, i.e., that the reason why the diagram is useful in proof is because of the insight we get from first seeing the same drawn line as the radius of a circle and then seeing it as one side of a triangle. In other words, some philosophers have tried to assimilate what happens in proofs like Euclid I,1 with Wittgenstein's famous drawing that can be seen both as a rabbit and as a duck. However, both cases are radically different. In seeing Wittgenstein's drawing as a picture of a duck, one must assign an interpretation to the drawing such that certain part of it represents its beak, another its eye, etc. If we want to see it then as a picture of a rabbit, we must see the part that represented the duck's beak as now representing the rabbit's ears, and so on. Abandoning one interpretation is necessary, before adopting the new one, because both interpretations are inconsistent. Nothing can be both a rabbit ear and part of a duck's beak. However, nothing of the sort is necessary in interpreting the diagram associated with Euclid I,1, for there is nothing inconsistent in a line being both the side of a triangle and the radius of a circle. After all, both the property of "being the side of a triangle" and the property of "being the radius of a circle" are extrinsic, relational properties, i.e., properties that a line has not because of any of its inherent features but because how it is related to other geometrical objects. As Emily Grosholz has emphasized, "Because the side of a triangle is a line, it is intelligible independent of the triangle, despite the fact that regarded as a side it is intelligible only in relation to the triangle as a whole." (Grosholz 2007, 36) Nevertheless, being related in one way to a geometrical object does not preclude the same line from being related in different ways to other geometrical objects. Thus, in explaining how we are able to regard line AB both as the radius of a circle and as one side of a triangle, there is no need to appeal to "seeing as". In order to make sense to the double role the line plays in the proof, its is enough to notice that when we describe AB as a radius, we pay attention to some of its extrinsic features, while we focus on different extrinsic features when we describe it as part of a triangle. In Macbeth's words (2014), "it is the shift in one's perceptual focus that effects what we would otherwise think of as a step in reasoning." When we regard it as a radius, we focus on its relation to the circle that has A as its center; when we regard it as the side of a triangle, we focus on its relation to lines BC and AC. There is nothing here that would make us abandon the thesis that diagrams are pictorial icons. Yes, Macbeth is right in asserting that, in Euclidean Geometry, "what a given line means is a function of how it is regarded in relation to other parts of the diagram" (2011, 69), but this is completely consistent, and actually expected, when dealing with pictorial icons, in general.

7. Conclusions

In this paper I have tried to offer an account of diagrams grounded in two main principles about our general use of representations. The first one is that representations used to make inferences are shaped by both informational and cognitive constraints, and diagrams are not an exception, The second one is that diagrams are pictorial icons and as such they exploit perceptual resemblance to fix their reference. I have tried to show how combining these two insights can throw some new light on some of their otherwise puzzling questions, like why the same diagram can be used in different contexts to represent different things, how do text and diagram interact in proof, and what does the diagram in a reductio proof represents. I have tried to show that as a consequence of the informational and cognitive constraints that shape diagrams, their visual resemblance to what they represent is usually not complete but partial, and this results in an underdetermination of their reference. In other words, I have tried to show why, in diagrams, just as in icon in general, visual resemblance constraints but does not fully determine reference. Most times, the diagram will resemble more than one different state of affairs, and we will need extra information to identify the intended referent among them. Thus, it is necessary to combine the information we perceive in the diagram with the information from the accompanying text to determine the content of a diagram. This allows for a more dynamic and malleable use and interpretation of diagrams, as is manifested in Euclidean proofs where the same feature of a diagram is used to represent different geometrical objects like Euclid's I.1. or to reduce a hypothesis to contradiction as on Euclid's I.6.

The account of the role of diagrams in Euclidean proof that I have presented so far takes as starting point the recognition that the representations we use to draw inferences about the world are shaped by two constraints: to include as much relevant information in the representation as possible with as little noise as possible, and to make the representation as easy to make, manipulate and interpret as possible. I have assumed, following a growing body of empirical evidence (Maes 2004, Paranobi 2007, Arias Trejo 2010, Jonson 2011), that one way to make a representation easy to interpret is by making it similar to what it represents. However, many times, making a representation resemble its referent also has its costs: while making the representation easy to interpret, it can make the representation difficult to produce. It can also introduce noise, i.e., it can add extra information that might not be relevant or true about the represented object. If there is relevant information that we cannot or better not reproduce in our representation, we have to incorporate it some other way. The most common way is by adding an accompanying text (Macbeth 2009). Similarly, if the representation includes false information, this is also something we can fix in the accompanying text. But then, we have two different sources of information about the relevant subject and this opens the possibility of inconsistencies between them. When inconsistencies occur, they can be resolved appealing to general pragmatic principles. They may be resolved by rejecting some of the information contained in the diagram or by rejecting some of the information contained in the text (Euclid's reduction proof of I.6 is an example). This explains why we can use the same diagram to represent different things in different contexts and even why we can use a diagram in a *reductio* proof without having to postulate impossible objects.

References

1. Abell, C. (2009) Canny Resemblance, *The Philosophical Review*, 118, 183-223.
2. Abell, C.(2005) Pictorial Implicature, *The Journal of Aesthetics and Art Criticism*, 63, 55-66.
3. Arias Trejo, N. and K. Plunkett, (2010), The effects of perceptual similarity and category membership on early word-referent identification, *Journal of Experimental Child Psychology,* 105, 63-80.
4. Bantinaki, K. (2008) The Opticality of Pictorial Representation, *The Journal of Aesthetics and Art Criticism* 66 , 183-191.
5. Barceló, Axel (*2016*) "Pictures as Epistemic Tools" *Sciential Studiae*
6. Blackwell, A.. (2008) Cognitive Dimensions of Notations: Understanding the Ergonomics of Diagram Use, in G. Stapleton, J. Howse and J. Lee (eds.) *Diagrammati Representation and Inference*, Berlin: Springer, 5-8.
7. Blumson, B. (2009) Images, Intentionality and Inexistence, *Philosophy and Phenomenological Research* 79, 522- 538.
8. Brown, J. R. (2008) *Philosophy of Mathematics: An Introduction to the World of Proofs and Pictures*, second edition, New York: Routledge.
9. Calderola, E. (2010) Pictorial Representation and Abstract Pictures, *Proceedings of the European Society for Aesthetic*s 2, 46-61.
10. Canales, J. (2009) *A Tenth of a Second,* Chicago The University of Chicago Press.
11. Dilworth, J. (2008) The Abstractness of Artworks and its Implications for Aesthetics, *The Journal of Aesthetics and Art Criticism*, 66, 341-353.
12. Giaquinto, M. (2007). *Visual Thinking in Mathematics*. Oxford Oxford University Press.

13. Giardino, V., (2012) Towards a diagrammatic classification, *Knowledge Engineering Review* 28, 237 – 248.
14. Grice, H. P., (1975) Logic and conversation, in P. Cole & J. Morgan (eds.) (1975) *Syntax and Semantics, 3 Speech Acts*, New York Academic Press. Reprinted in H. P. Grice (1989) *Studies in the Way of Words*, Cambridge, MA Harvard University Press, 22–40.
15. Hyman, J. (2006) *The Objective Eye Colour, Form and Reality in the Theory of Art,* Chicago and London: The University of Chicago Press.
16. Johnson, E. K., J. McQueen and F. Huettig, (2011), "Toddlers language-mediated visual search: They need not have the words for it", *The Quarterly Journal of Experimental Psychology*, 64, 1672-1682.
17. Kripke, S. (1980) *Naming and Necessity*, Harvard University Press.
18. Krummheuer, G. (2009) Inscription, Narration and Diagrammatically Based Argumentation: Narrative Accounting Practices in Primary School Mathematics Lessons in Wolff-Michael Roth (ed.) *Mathematical representation at the interface of body and culture*, Charlotte, North Carolina: Information Age Publishing, 219-244.
19. Kulpa, Z. (2009) Main Problems of Diagrammatic Reasoning. Part I: The Generalization Problem, Foundations of Science 14, 75-96.
20. Kulvicki, J. (2010) Knowing with Images: Medium and Message, *Philosophy of Science* 77, 295-313.
21. Larkin, J. H. and H. A. Simon, (1995) Why a diagram is (sometimes) worth ten thousand words, in B. Chandrasekaran, J. Glasgow and N. Hari Narayan (eds.) (1995) *Diagrammatic Reasoning: Cognitive and Computational Perspectives*, Boston: AAAI Press: The MIT Press, 69-109.
22. Lomas, D. (2002) What Perception is Doing, and What It is Not Doing, in Mathematical Reasoning, *British Journal for the Philosophy of Science* 53, 205-223.
23. Macbeth, D. (2014) *Realizing Reason: A Narrative of Truth and Knowing*, Oxford: Oxford University Press.
24. Macbeth, D.(2010) Diagrammatic reasoning in Euclids Elements, in B. Van Kerkhove, J. De Vuyst, and J.-P. Van Bendegem (eds.), *Philosophical perspectives on mathematical practice*, London: College Publications, 235–267.
25. Maes, A., A. Arts and L. Noordman, (2004) Reference Management in Instructive Discourse, *Discourse Processes* 37, 117-144.
26. Manders, K. (2008) The euclidean diagram, in P. Mancosu (editor), *The Philosophy of Mathematical Practice*, Oxford: Oxford University Press, 112-183.
27. Mumma, J. (2010) Proofs, pictures and Euclid, *Synthese* 175, 255-287.
28. Netz, R. (1998) Greek Mathematical Diagrams: Their Use and Their Meaning, *For the Learning of Mathematics,* 18, 33-39.
29. Norman, J. (2006) *After Euclid: visual reasoning & the epistemology of diagrams*, Stanford: CSLI Publications.
30. Panza, M. (2012) The twofold role of diagrams in Euclid's plane geometry, *Synthese* 186, 55-102.
31. Paraboni, I., K. van Deemter and J. Masthoff (2007) Generating Referring Expressions: Making Referents Easy to Identify, *Computational Linguistics* 33, 229-254.
32. Schier, F. (1986) *Deeper into Pictures*, Cambridge: Cambridge University Press.
33. Sherry, D. (2009) The Role of Diagrams in Mathematical Arguments, *Foundations of Science* 14, 59-74.
34. Shimojima, A. (1996) Reasoning with Diagrams and Geometrical Constraints, *Logic, Language and Computation*, 1, 527-540.