Marco Panza on

What sort of generality (if any) is involved in Euclid's geometric propositions?

For some years now, Marco Panza has been arguing that the answer to this question cannot be the traditional one, because there is no domain or concept binding together all the objects the proposition applies to previous to the proof and proposition. Even though we can talk about the *objects* Euclidean geometry is about, these objects do not form a domain in the traditional sense, i.e., one in which we can quantify into, one in which the objects have identity.

Furthermore, there is no (neither internal nor external) way to differentiate between the members of the domain. Their identity is only local.

One can say that, given two mathematical objects A and B, the only difference between A and B is that one is A and the other is B. But even saying this is an understatement, for when we say "given two lines A and B", we cannot even tell that they are two and not one!

If we want to think of Euclidean Geometry as a sort of deductive system like natural deduction, as Macbeth and Mäenpää and von Pla have argued, then instead of allowing us to deduce truths from truths, it would be better to say that it allows us to deduce GIVEN from GIVEN — that is why it does not deal with propositions but geometrical objects. Panza borrows from Taisbak the passive/active terminology to describe what would otherwise be the premises/conclusions distinction. The objects given in the setting of the problem are passively given and from them, we construct the actively given ones.

In arithmetic, we start from particular numbers *–7, 1987, 200, 2,345,999,* etc. –, and study their properties; this does not seem to be what is happenning in geometry. When we draw a triangle in a geometric diagram, the triangle we draw is a particular one, but its reference is not: there is no such thing as the triangle it represents, even though it is true that it represents <u>*a*</u> traingle. It has a length, but no particular one. It has a position, but no particular one. Still, it has some definite particular properties. For example, the number of sides and the number of angles is determinate and very particular: Three. Three is a particular specific number, thus it would be a mistake to say that this triangle only has indeterminate properties.

Panza is worried about how could geometrical objects have saptial properties if they are abstract? So, he concludes, the cannot be *fully* abstract, they have to be quasi-concrete in so far as some of their properties are the kind of properties that concrete and not abstract objects have, i.e., spatial properties. Then, he focuses on how we come to know these properties of these objects. It cannot be by analyzing their definition, as Kant has argued; so some form of spatial intuition needs to be appealed to. This is where diagrams come in. They provide the spatial intuition that gives content to our proofs about geometrical objects. This, however, is not actually perceptual, in so far as it invovles some kind of interpretation — what Panza calls 'taking' the entities that constitute the diagrams as being taken to have certain attributes.