

Marco Panza's Twofold Role of Euclidean Diagrams

Euclidean geometry is about (a certain sort of) geometrical objects, i.e., classes of equivalence of configurations of points and lines. In general, for a method to appropriately help us achieve knowledge about a certain domain of objects, such method must be appropriately related to those objects. Thus, diagrams can only be part of a reliable method for knowing about geometric objects if they are properly related to them. We know that this 'properly related' condition need not be causal, it can also be representational, and thus Panza argues that diagrams are representation of geometrical objects. But, of course, the notion of representation is quite vague and needs to be further developed and specified in order to be adequately explanatory. Thus, he rules out certain sort of relations that could be considered representational but do not fill the role of diagrams in Euclidean Geometry. For example, they do not depict, mimitize or are tokens of what they represent. For Panza, they are symbols (neither icons, nor indexes)! Because what is important about diagrams is not what we perceive them to be, but what we **take them** to be in accordance to explicit syntactic and semantic compositional rules. "What matters here, is neither how the concrete lines representing the relevant circles are, nor how they are required to be, but rather how they are taken to be." (Panza 2012, 74)

Panza calls his account a kind of Kantianism (more than an Aristotelism) because he takes that neither the rules of Euclidean argumentation and diagramming, nor the diagrams as drawings are enough, on their own, to deliver all the mathematical knowledge contained in Euclidean Geometry. Geometrical objects have some attributes – what he calls *diagrammatica* attributes – that are neither given in the rules, nor can be just seen in the diagrams, but are grasped through the interaction of drawing and rules. From the rules, we get how the drawing should be taken (aka interpreted, in my terms); this means that the drawing by itself does not tell us what it represents, and which of the attributes of the drawing correspond to attributes of the objects they represent. Rules are not enough because there are objects and attributes that only *pop up* – to use Macbeth's term – when the construction is actually (even if only in the imagination) is carried out.

"Kant maintains that mathematical judgments are synthetic – that we cannot ground them merely through reflection on their constituent concepts. Instead, he argues, we must construct those concepts, i.e., "exhibit a priori the intuition which corresponds to" them, grounding our judgments on what can be made evident only through such construction. (A 713/B 741)" (Young 1992, p 159)

Yet, this does not explain why Euclidean diagrammatic argumentation is reliable about geometrical objects. Here is where the Twofold Role of Euclidean Diagrams that Panza alludes to in his title comes in. Diagrams determine the geometrical object's identity and some of its attributes. In this sense, the objects of Euclidean Geometry are very much like the characters of a work of fiction: they only have local identity, i.e., there is only an answer to the question "is *A* the same geometrical object as *B*" when they are represented by syntactic/semantic components of the same diagram. This is just like we can only answer the question whether "is *A* the same fictional character as *B*" when they are represented by syntactic/semantic components of the same work of fiction. Thus, the diagram is reliable about the identity of its referent. Something similar can be said about some attributes, since the objects inherit them from the diagrams, the diagram is a reliable indicator that the objects have them. These two features (roles) of diagrams in Euclidean Geometry ground their epistemic reliability.

In other words, in Panza's interpretation, Euclidean diagrammatic arguments are neither empirical, nor rational; Euclidean objects are neither Aristotelean abstractions from concrete instances nor formal objects completely given in the definitions.