

Contents

Copyright	ii
Acknowledgements	iv
Abstract	vi
Contents	viii
1. Introduction	1
I. Wittgenstein's Philosophy of Mathematics during the Middle Period	5
A. Wittgenstein	5
B. Wittgenstein and Mathematics	8
C. The Middle Period	13
II. Language, Grammar and Mathematics	17
2. On Mathematical Objects and Concepts	20
I. Introduction	20
II. Wittgenstein's Criticism of the Concept-Object Distinction	22
A. <i>Zahlangaben</i>	22
B. Frege's Grammatical Enquiry into the Concept of Number	25
C. The Grammatical Method	27
D. Wittgenstein's Criticism of Frege's Grammatical Analysis of <i>Zahlangaben</i>	28
E. Mathematical Objects and Concepts	31

III. On Mathematical Equations	35
A. Frege and Ramsey: Mathematical Equations as Identity Statements	35
B. Equations and Calculations	41
IV. An Extension of Frege's Context Principle	43
A. The Context Principle	43
B. Specification and Calculation Propositions and Concepts	43
V. Beyond Arithmetic, Proofs as Calculations	45
VI. Conclusion	48
3. The Grammar of Calculation	50
I. Introduction	50
II. Mathematics as Calculation	51
A. “There’s a <i>Fascination</i> here:” Mathematical Problems and Problems of Mathematical Investigation	51
B. Method and Solution	55
C. Calculations are Transitions between Expressions According to a Rule	56
III. Mathematical Correctness	58
A. Wittgenstein’s Anti-Platonism	59
1. Platonist Mathematical Explorations	59
2. Searches and Explorations	62
B. Correct Calculations and Correct Results	66
1. Calculations as Transitions	66
2. Traces	68
IV. Conclusion	70

4. Mathematical Application [<i>Anwendung</i>]	72
I. Introduction	72
II. Calculation's Role in the Solution of <i>Practical</i> Problems	75
A. Wittgenstein's Criticism of the Logicians' Account of Mathematical Application	77
1. The Logician's Puzzle	77
- 2. First Myth: Mathematics is the Most Universal of Sciences	78
3. Wittgenstein against the Universality of Mathematics	79
4. Letters and Schemes	80
5. Wittgenstein and Ramsey on <i>Anwendung</i> and Interpretation	84
B. Wittgenstein's Account of Mathematical Application	87
III. <i>Anwendung</i> and the Foundations of Mathematics	91
A. The Autonomy of Mathematical Calculi	91
B. <i>Anwendung</i> is an Essential Feature of Mathematics	94
C. On Consistency [<i>Wiederpruchsfreiheit</i>]	97
IV. Conclusion	99
5. Grammar	101
I. Introduction	101
II. A Formal Background for the Discussion of Wittgenstein's Grammar	102
A. Language	103
B. What is Grammar?	104
C. The Conventional Approach	108
D. Example: The Language of Propositional Calculus	114
1. The Language of Propositional Calculus	114

2. The Grammar of Propositional Calculus	114
E. Strong, Redundant and Trivial Grammars	119
1. Strong Grammars	120
2. Redundant Grammars	121
3. Trivial Grammars	124
III. Wittgenstein's Approach	124
IV. A comparison between the Two Approaches	126
A. Wittgenstein Grammar	126
B. Formal Requirements for a Wittgenstein Grammar	131
V. Conclusion	135
6. Mathematics as Grammar. A Formal Treatment	136
I. Introduction	136
A. A Formal Model of Grammar	136
B. <i>Anwendung</i> and the Grammar of Natural Language	137
C. Mathematics as Grammar	138
II. A Grammatical Analysis of the Internal <i>Anwendung</i> of Arithmetic	140
A. Introduction	140
1. Calculi as Languages	140
2. A Grammatical System of Natural Numbers	140
3. The Grammar of Addition	141
B. A Formal Grammatical Analysis of a Subsystem of the Arithmetic of Natural Numbers with Addition as its Single Operation	142
1. The Calculus as Language	142
2. Grammatical Number Theory	143
3. The Grammar of Addition	143

C. Mathematical Induction	148
III. A Grammatical Analysis of the External Application of Arithmetic	149
A. A Numerical System	150
1. ‘One’	150
2. ‘Two’	155
3. From ‘One’ to ‘Two’ and Beyond: Succession and Induction	155
B. Calculation: the Case of Division	160
IV. Conclusion: A False Dilemma	161
7. <i>Zahlangaben</i> Revisited	164
I. Introduction	164
II. Grammatical Concepts and Systems of Propositions	164
A. Systems of Propositions	164
1. Systems of Propositions Formally Defined	168
2. Complete Descriptions	170
3. Grammatical Concepts Formally Defined	171
B. Grammatical Concepts and Systems of Propositions	172
1. A Warning Notice	174
III. <i>Zahlangaben</i> Revisited	176
A. On the Extensionality of Specification Concepts	177
B. Arithmetical Propositions	181
IV. Conclusion	183
8. Grammatical Necessity	185
I. Introduction	185
II. On the Grammatical Nature of Mathematics	187

A. Arguments against the Grammatical Nature of Mathematics	187
B. Grammatical Statements	189
C. Grammatical Propositions as Rules	194
D. Grammar and Vocabulary	196
III. Wittgenstein's Syntactic Necessity and Analyticity	199
A. Brief Historical Background	199
B. Carnap	201
C. Quine	205
1. "Two Dogmas of Empiricism" and the Analytic Nature of Grammar	205
2. Convention and Justification	209
IV. Conclusion: Wittgenstein's Own Account of Analyticity	212
9. Bibliography	215